



S.S. PAPADOPULOS & ASSOCIATES, INC.
ENVIRONMENTAL AND WATER RESOURCE CONSULTANTS

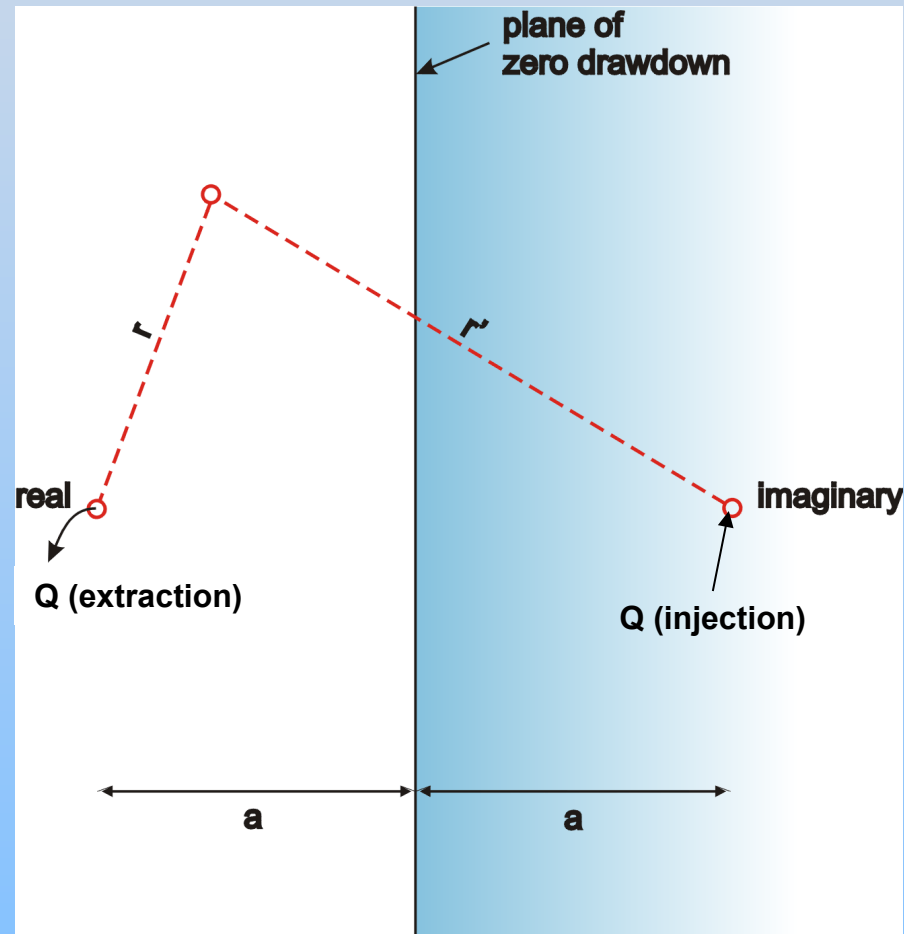
The Interpretation of Pumping Tests in Aquifers with Linear Boundaries

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Simplified conceptual models

1. Linear constant-head boundary
2. Linear no-flow boundary
3. Two linear no-flow boundaries (strip)
→ Buried channel aquifers

Model 1: Linear, constant-head boundary



Theis solution with an image well:

$$s(r, t) = \frac{Q}{4\pi T} W\left(\frac{r^2 S}{4Tt}\right) - \frac{Q}{4\pi T} W\left(\frac{r'^2 S}{4Tt}\right)$$

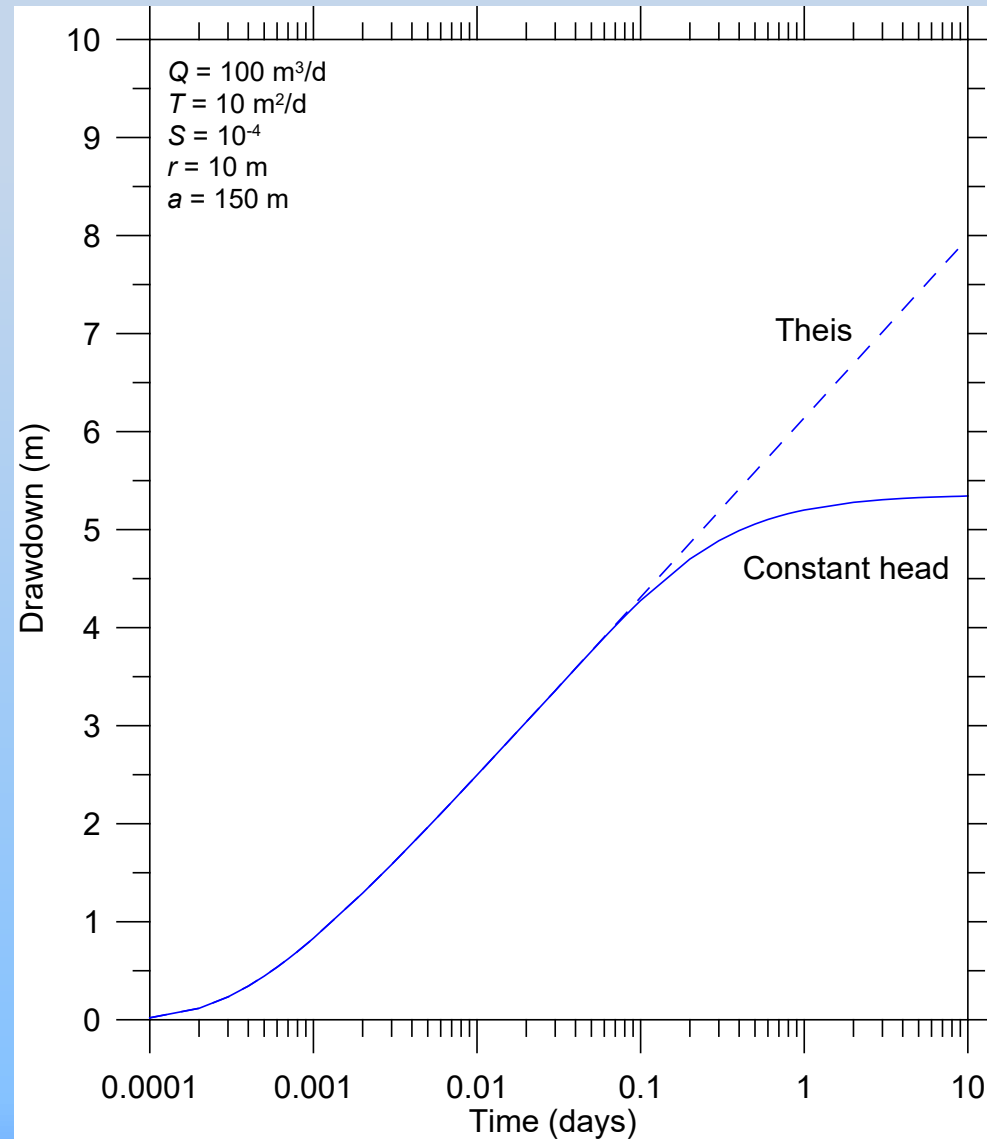
Cooper-Jacob approximation:

$$s(r, t) \approx \frac{Q}{4\pi T} \left[-0.5772 - \ln \left\{ \frac{r^2 S}{4Tt} \right\} \right] - \frac{Q}{4\pi T} \left[-0.5772 - \ln \left\{ \frac{r'^2 S}{4Tt} \right\} \right]$$

Cooper-Jacob approximation (2):

$$\begin{aligned} s(r, t) &\approx \frac{Q}{4\pi T} \left[-0.5772 - \ln \left\{ \frac{r^2 S}{4Tt} \right\} \right] - \frac{Q}{4\pi T} \left[-0.5772 - \ln \left\{ \frac{r'^2 S}{4Tt} \right\} \right] \\ &= \frac{Q}{4\pi T} \left(\left[-\ln\{r^2\} - \ln \left\{ \frac{S}{4Tt} \right\} \right] + \left[\ln\{r'^2\} + \ln \left\{ \frac{S}{4Tt} \right\} \right] \right) \\ &= \frac{Q}{4\pi T} \ln \left\{ \frac{r'^2}{r^2} \right\} \end{aligned}$$

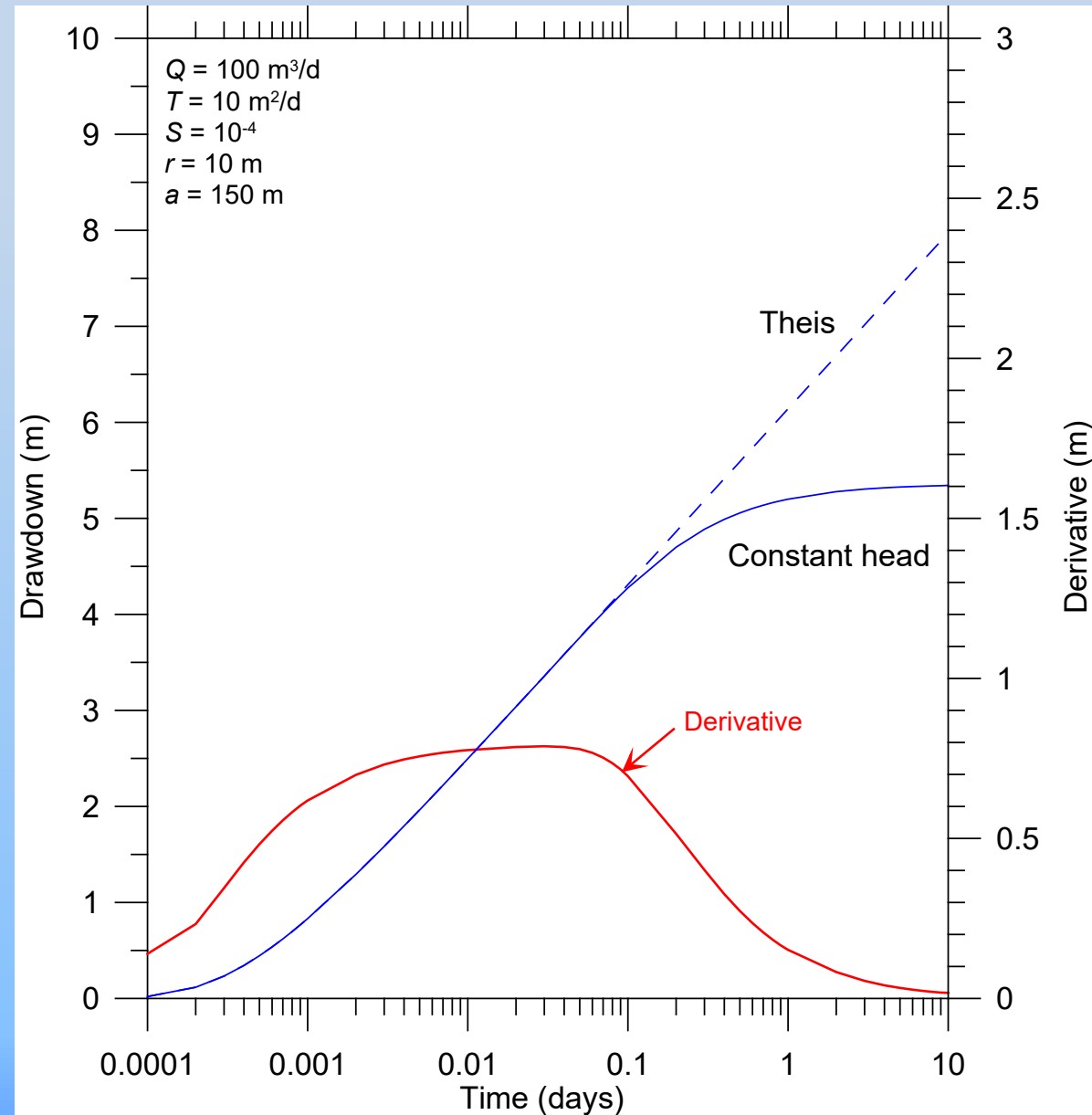
Semi-log plot



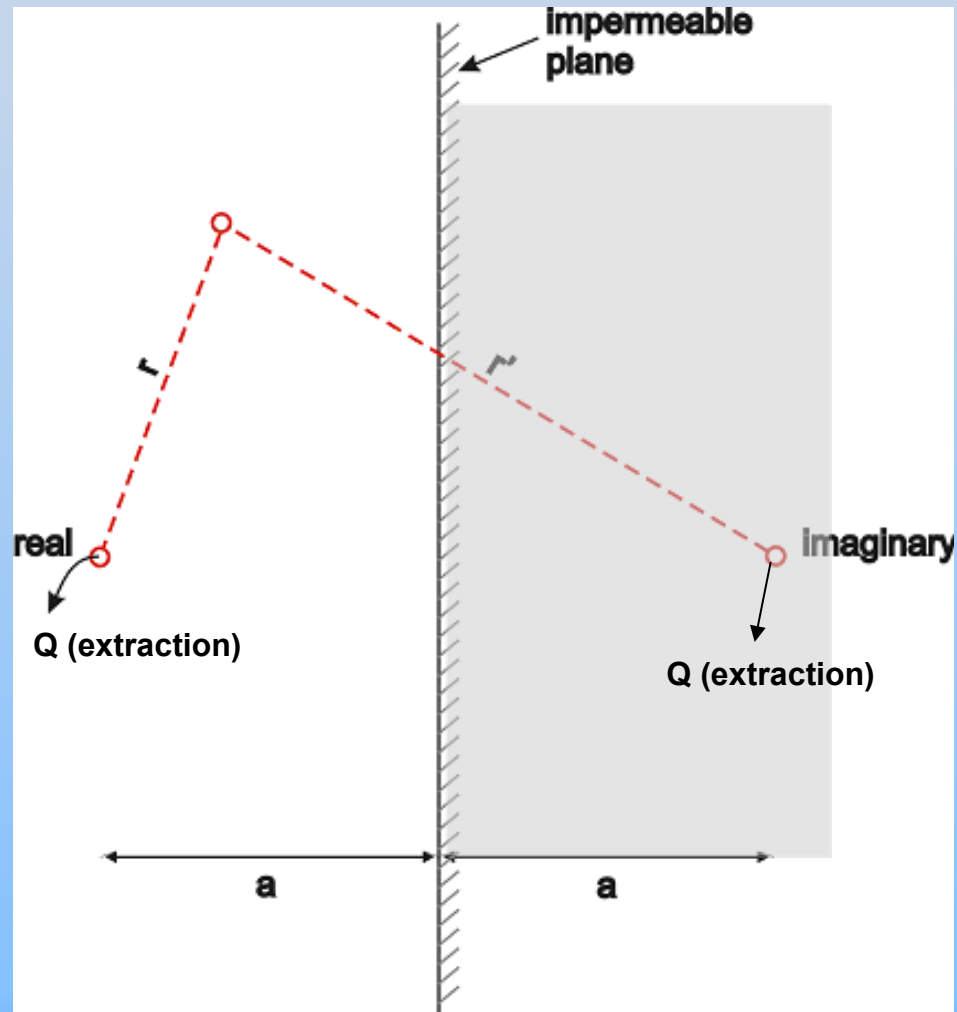
Late-time derivative

$$\begin{aligned} D_t(s) &= \frac{\partial s}{\partial(\ln\{t\})} \\ &= \frac{\partial}{\partial(\ln\{t\})} \left[\frac{Q}{4\pi T} \ln \left\{ \frac{r'^2}{r^2} \right\} \right] \\ &= 0 \end{aligned}$$

Semi-log plot with derivative



Model 2: Linear, impermeable boundary



Theis solution with an image well:

$$s(r, t) = \frac{Q}{4\pi T} W \left(\frac{r^2 S}{4Tt} \right) + \frac{Q}{4\pi T} W \left(\frac{r'^2 S}{4Tt} \right)$$

Cooper-Jacob approximation:

$$s \approx \frac{Q}{4\pi T} \left[-0.5772 - \ln \left\{ \frac{r^2 S}{4Tt} \right\} \right] + \frac{Q}{4\pi T} \left[-0.5772 - \ln \left\{ \frac{r'^2 S}{4Tt} \right\} \right]$$

Cooper-Jacob approximation ...

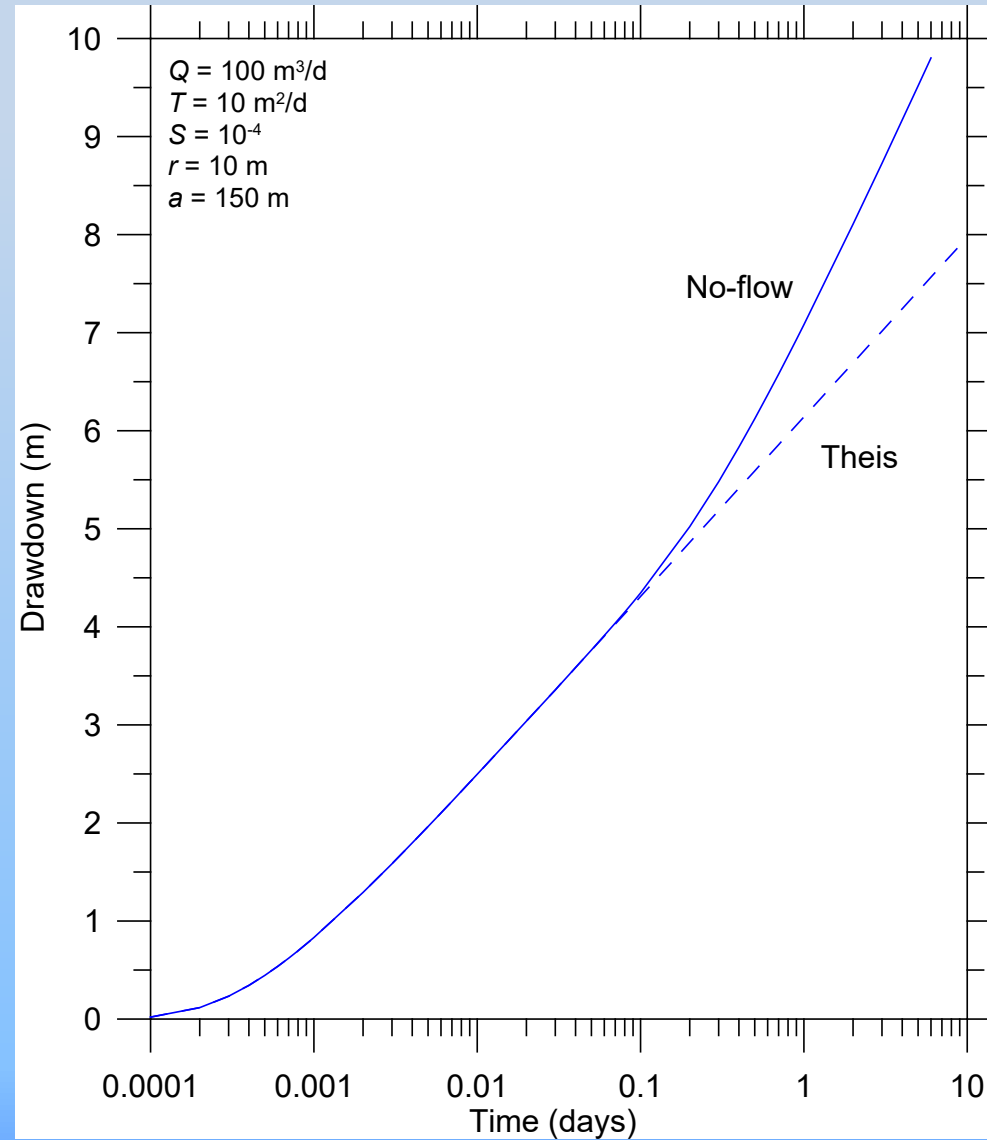
$$s \approx \frac{Q}{4\pi T} \left[\left(-0.5772 - \ln \left\{ \frac{r^2 S}{4Tt} \right\} \right) + \left(-0.5772 - \ln \left\{ \frac{r'^2 S}{4Tt} \right\} \right) \right]$$

$$= \frac{Q}{4\pi T} \left[2(-0.5772) - 2 \ln \left\{ \frac{S}{4Tt} \right\} - 2 \ln\{r\} - 2 \ln\{r'\} \right]$$

$$= \frac{Q}{4\pi T} \left[2 \ln\{0.561\} + 2 \ln \left\{ \frac{4Tt}{S} \right\} + 2 \ln \left\{ \frac{1}{rr'} \right\} \right]$$

$$= \frac{Q}{2\pi T} 2.303 \left(\log_{10} \left\{ 2.2459 \frac{T}{S} \frac{1}{rr'} \right\} + \log_{10}\{t\} \right)$$

Semi-log plot

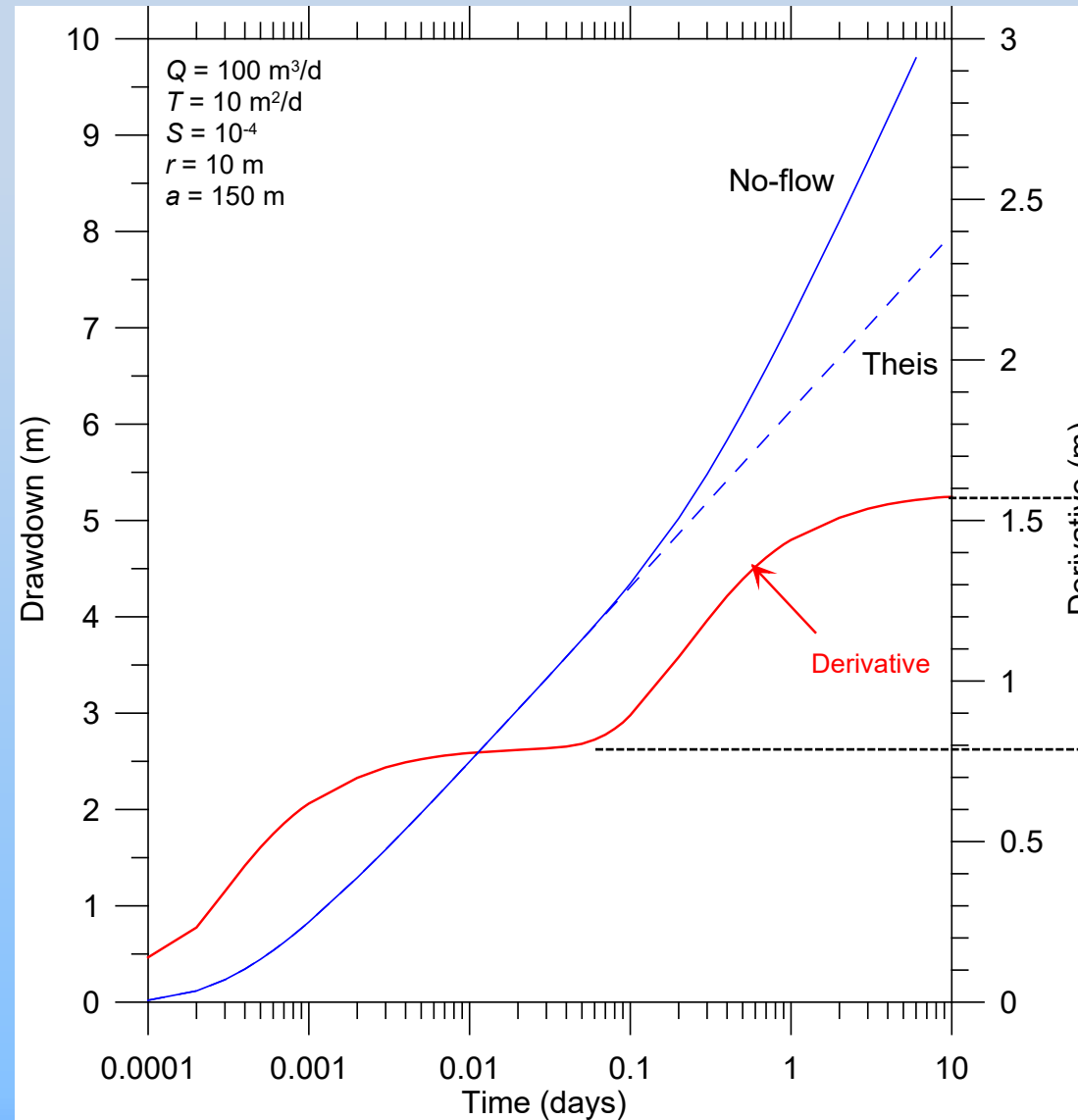


Late-time derivative

$$\begin{aligned} D_t(s) &= \frac{\partial s}{\partial(\ln\{t\})} \\ &= \frac{\partial}{\partial(\ln\{t\})} \left[\frac{Q}{2\pi T} \ln \left\{ \left[2.2459 \frac{Tt}{S} \right] \frac{1}{rr'} \right\} \right] \\ &= \frac{Q}{2\pi T} \end{aligned}$$

$$D_t = 2 \times \frac{Q}{4\pi T}$$

Semilog plot with derivative

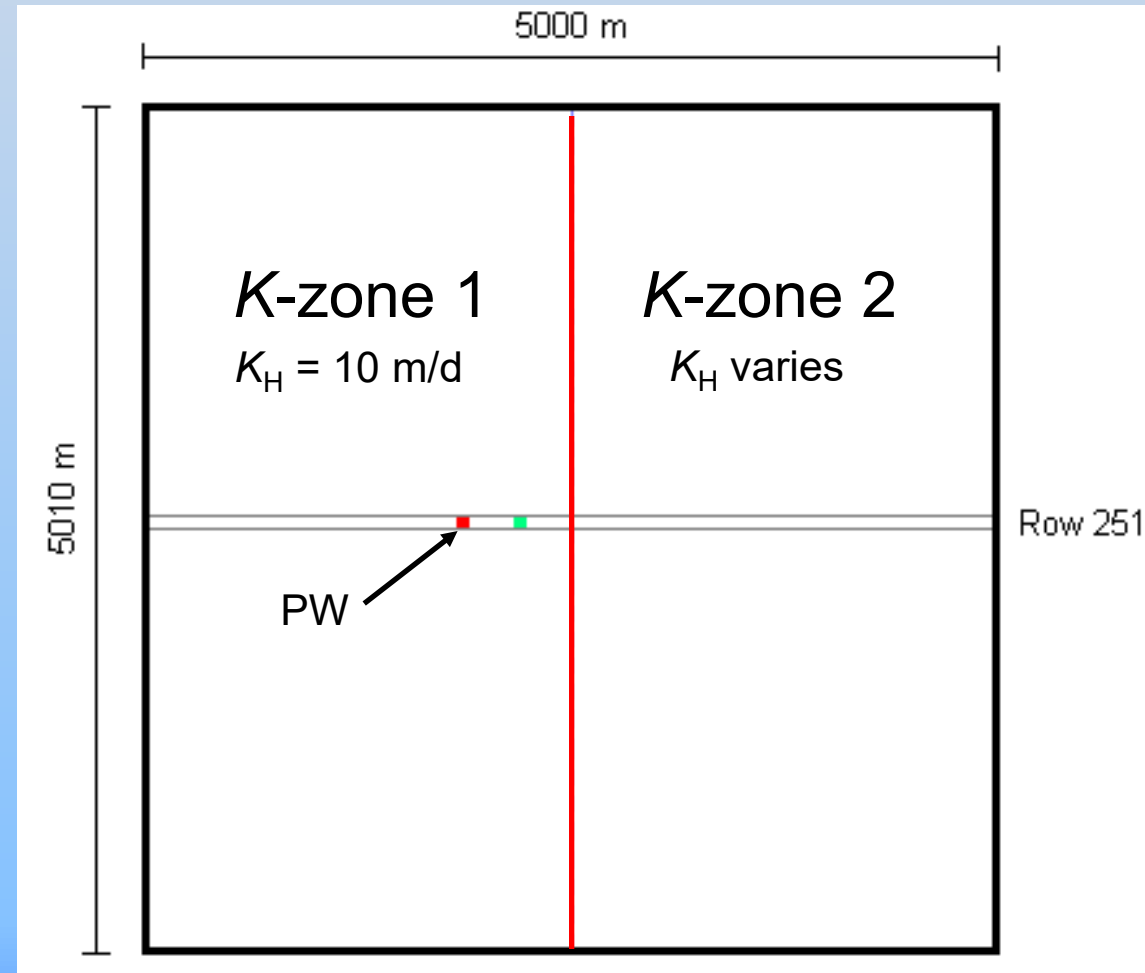


$$D_t = 1.56 \text{ m}$$

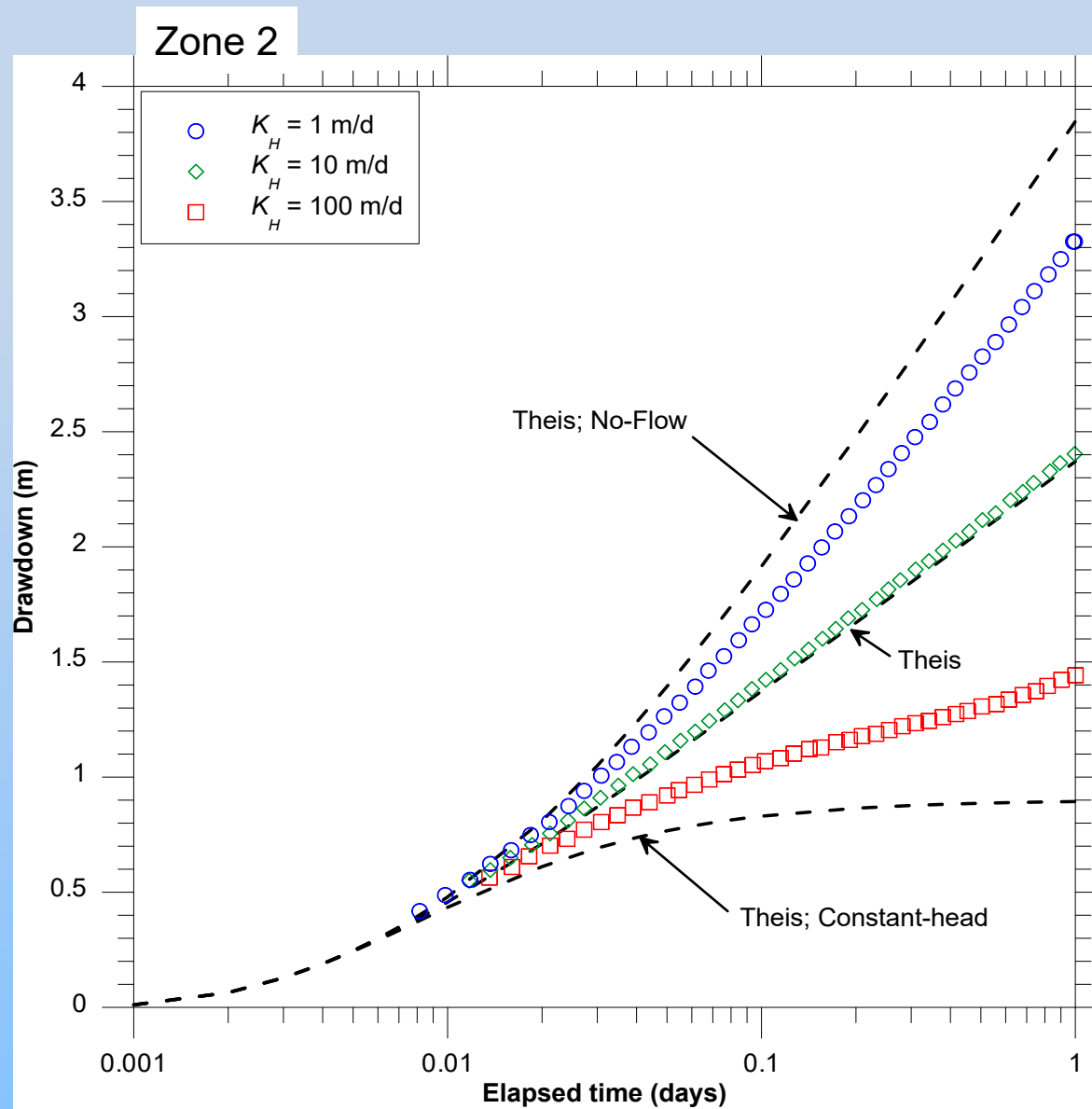
$$D_t = 0.78 \text{ m}$$

Diagnosis of an “Imperfect” linear boundary

Example analysis for a 2-zone aquifer



Zone 1: $K_H = 10$ m/d

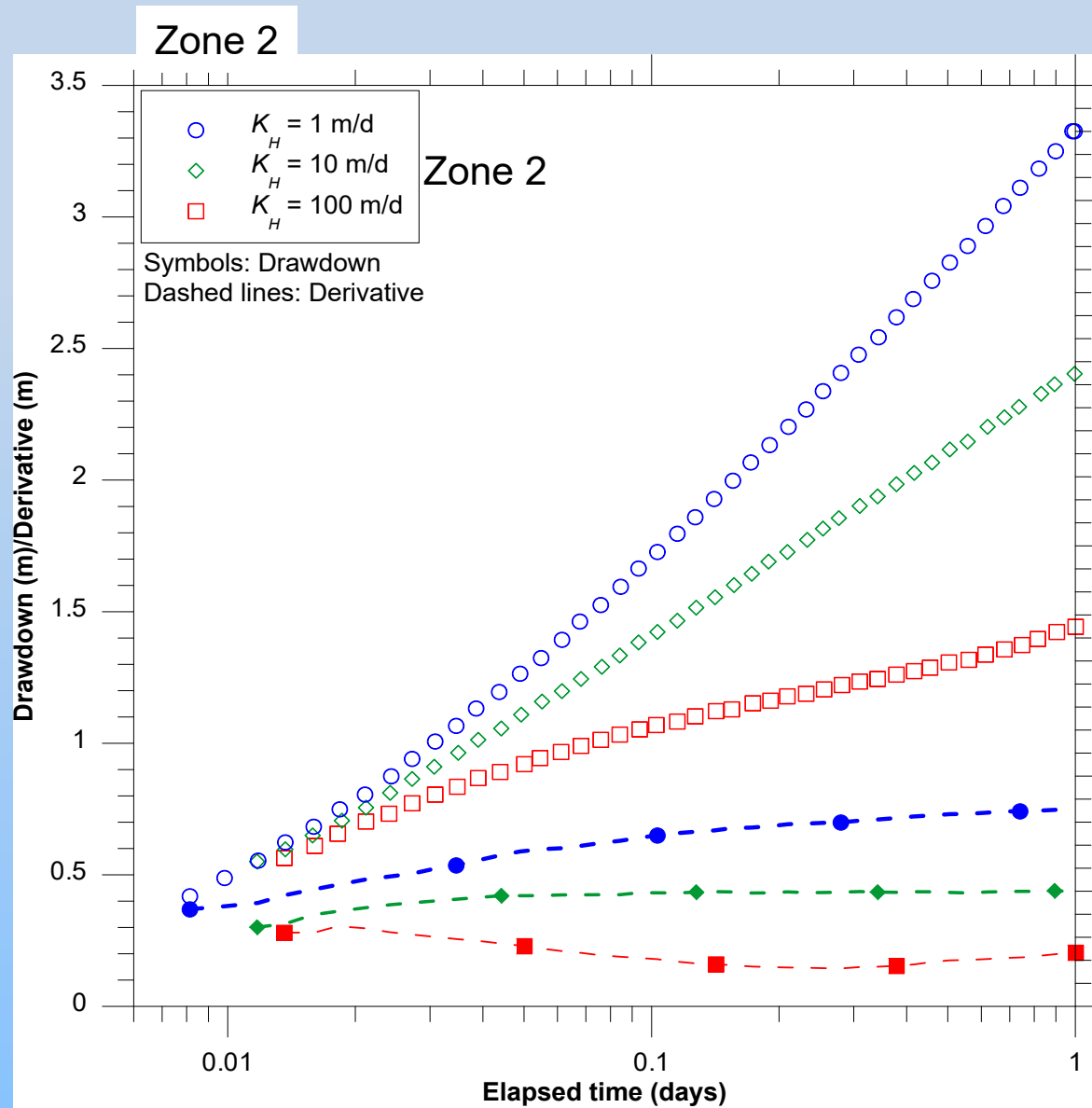


Drawdown

$K_2 < K_1$

$K_2 > K_1$

Zone 1: $K_H = 10$ m/d

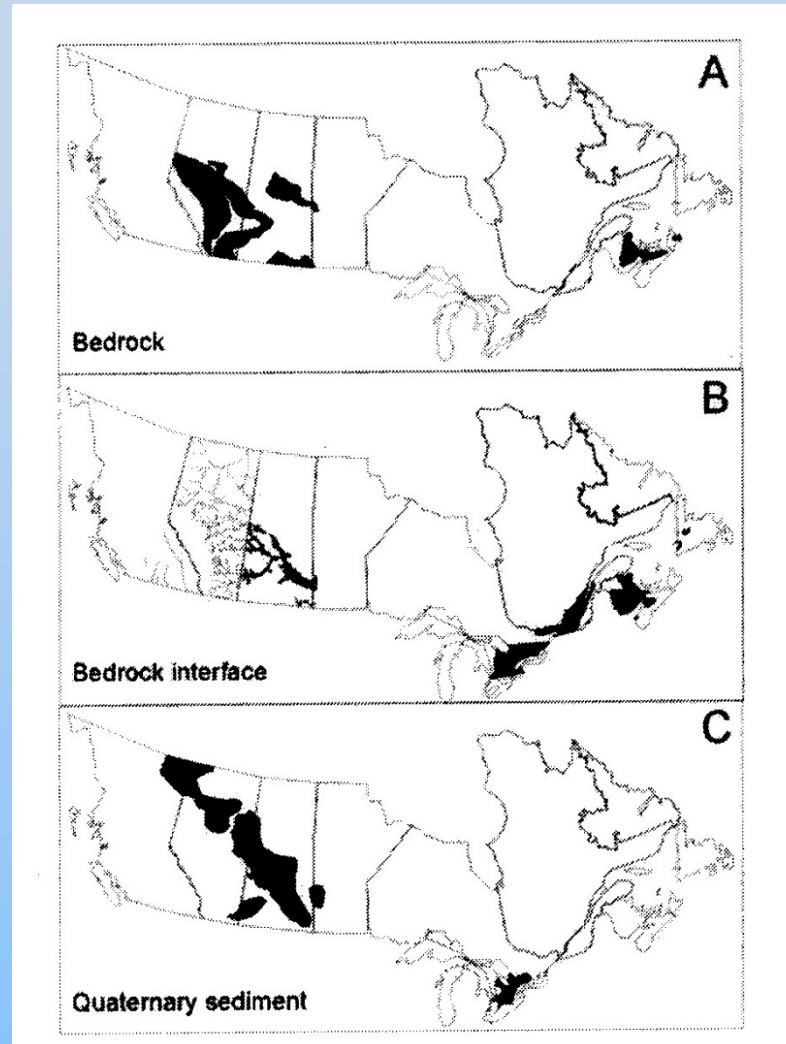


Derivative

$K_2 < K_1$

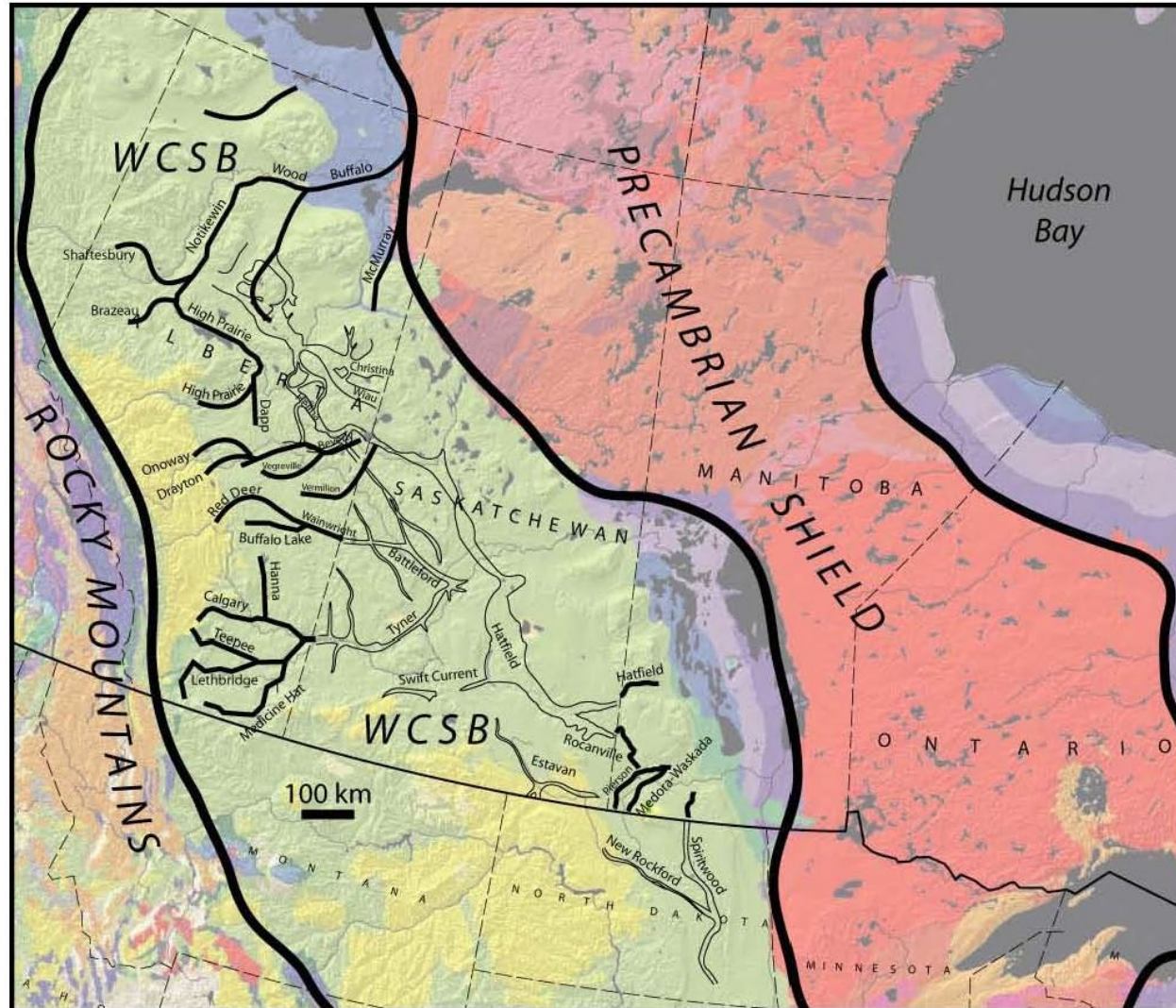
$K_2 > K_1$

Analysis of pumping tests in buried channel aquifers

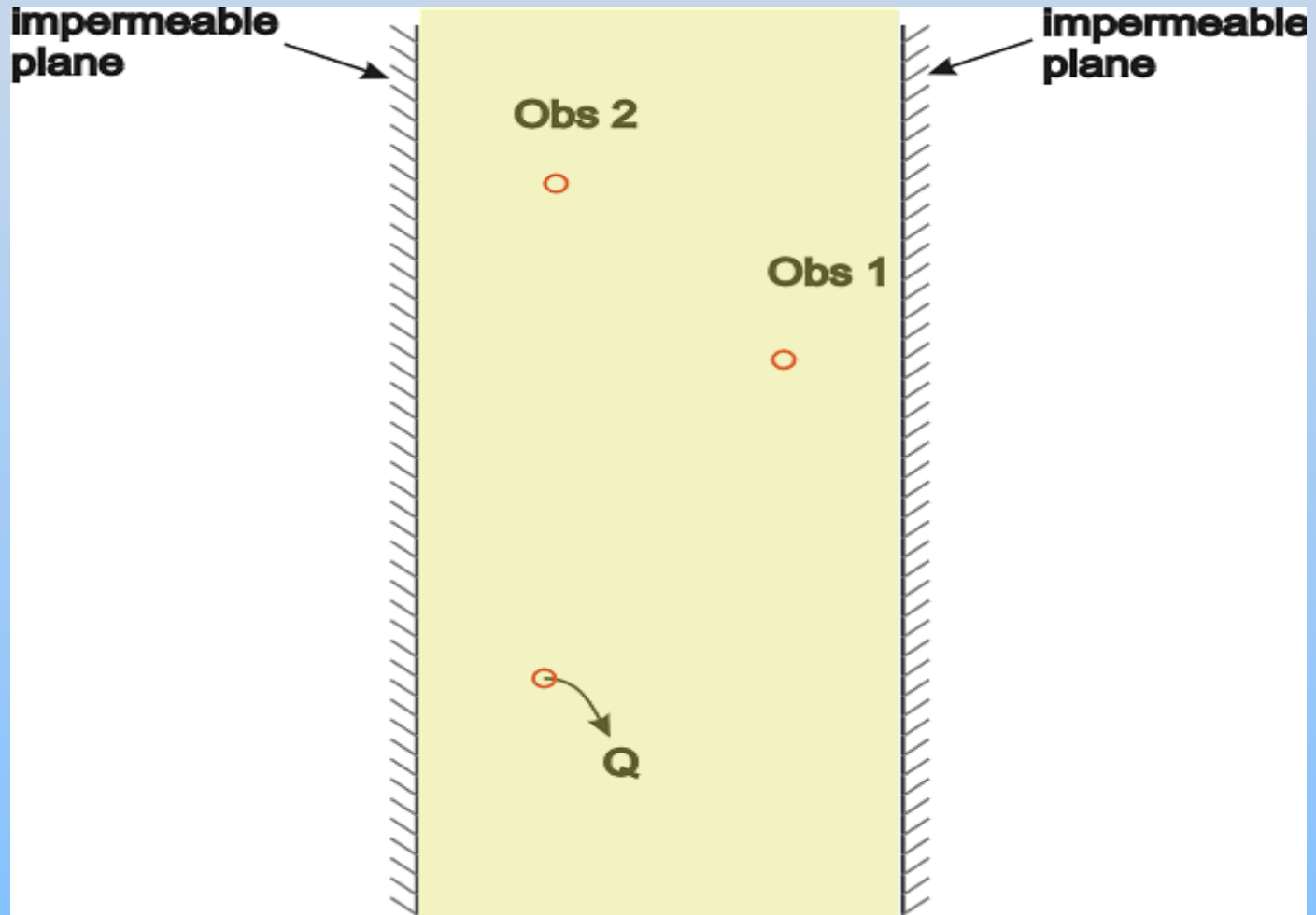


Source: Russell, Hinton, van der Kamp, and Sharpe, 2004

Buried valley aquifers



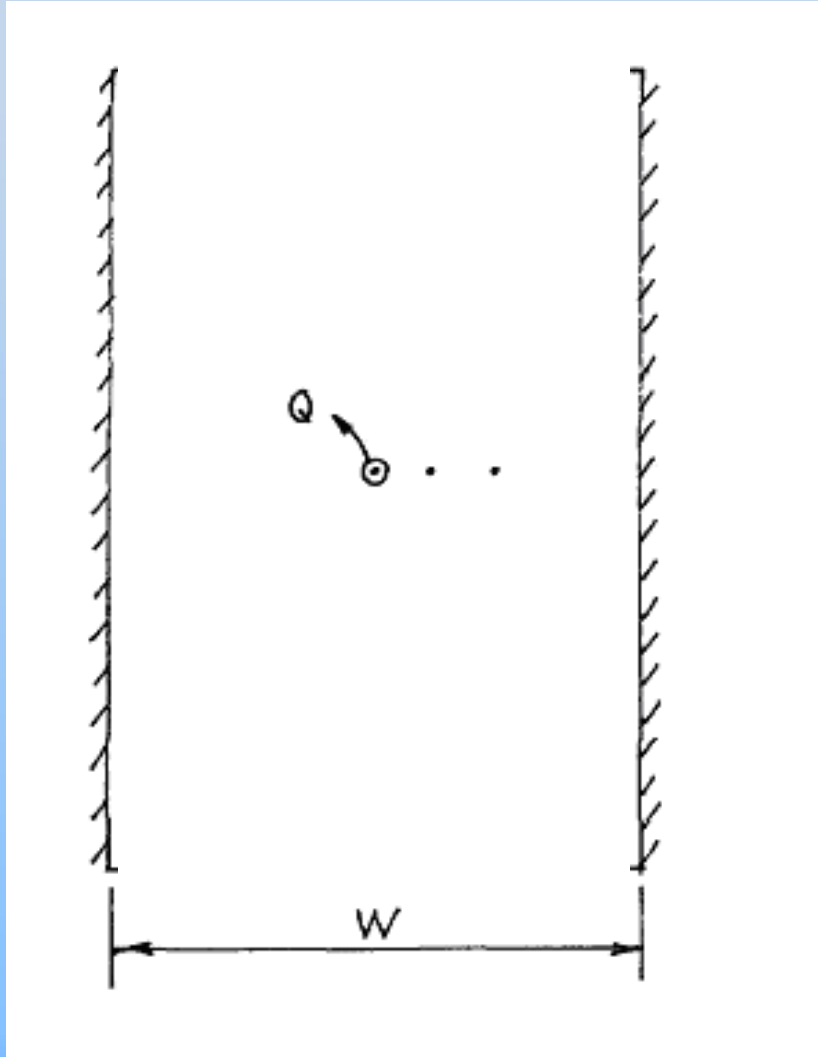
Idealized model for a buried channel aquifer



The two most important aspects of the analysis of a buried channel aquifer

- Recognition that the aquifer is a buried channel
- Identification of the portion of the response for the estimation of the transmissivity, that is, the portion that is not affected by the boundaries

Example analysis



$$T = 8.64 \text{ m}^2/\text{d}$$

$$S = 1.0 \times 10^{-4}$$

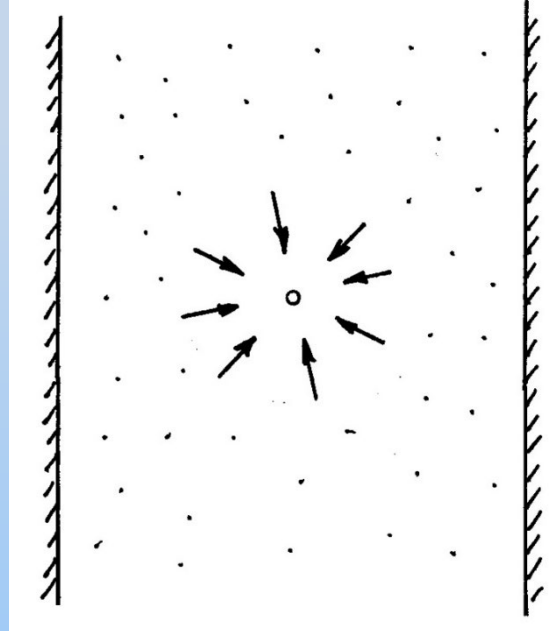
$$W = 510 \text{ m}$$

$$x_{OW1} = 20 \text{ m}$$

$$x_{OW2} = 50 \text{ m}$$

Early time response

Radial flow



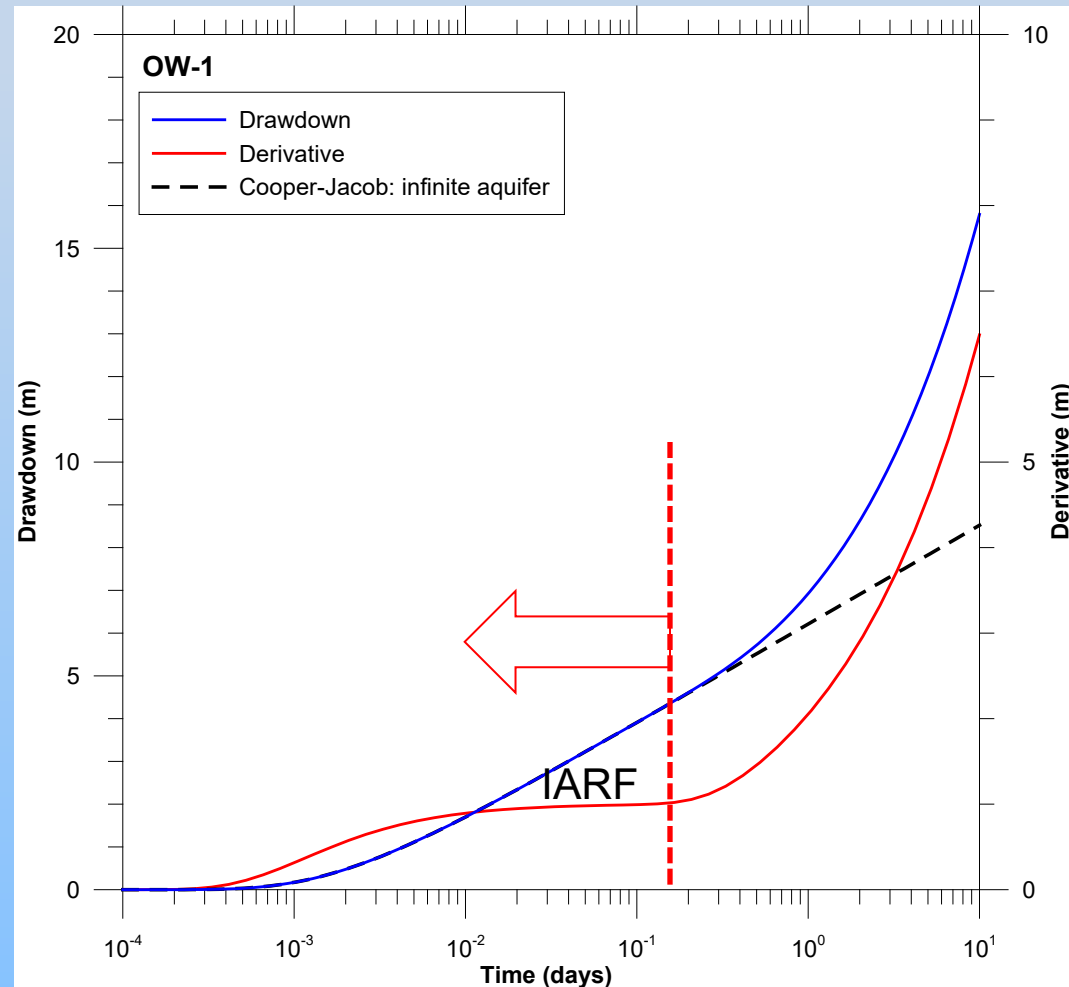
$$s(r, t) = \frac{Q}{4\pi T} W\left(\frac{r^2 S}{4Tt}\right) \cong \frac{Q}{4\pi T} \left[-0.5772 + \ln \left\{ \frac{4Tt}{r^2 S} \right\} \right]$$

$$\rightarrow s(r, t) \sim \log\{t\}$$

$$D_t \rightarrow \text{constant}$$

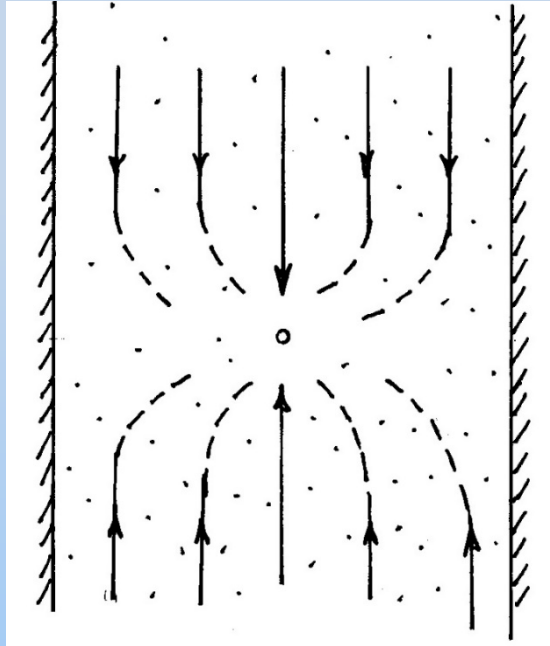
Estimation of transmissivity (pre-boundary effects)

Semilog plot with derivative: Early time plateau



Late time response

Linear flow



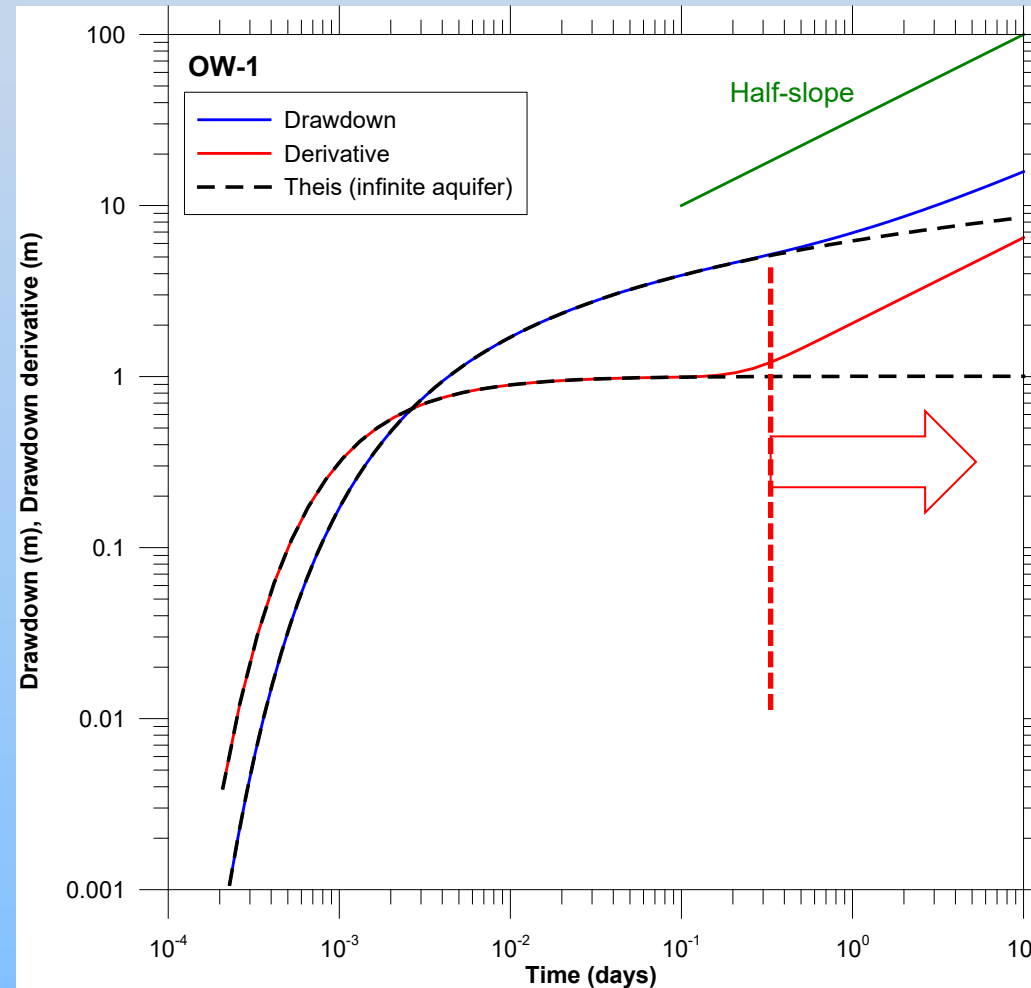
$$s(r, t) \cong \frac{Q}{W} \frac{1}{2T} \left[\left(\frac{4Tt}{\pi S} \right)^{1/2} - y \right]$$

$$s(r, t) \sim t^{1/2} \rightarrow \log[s] \sim \frac{1}{2} \log[t]$$

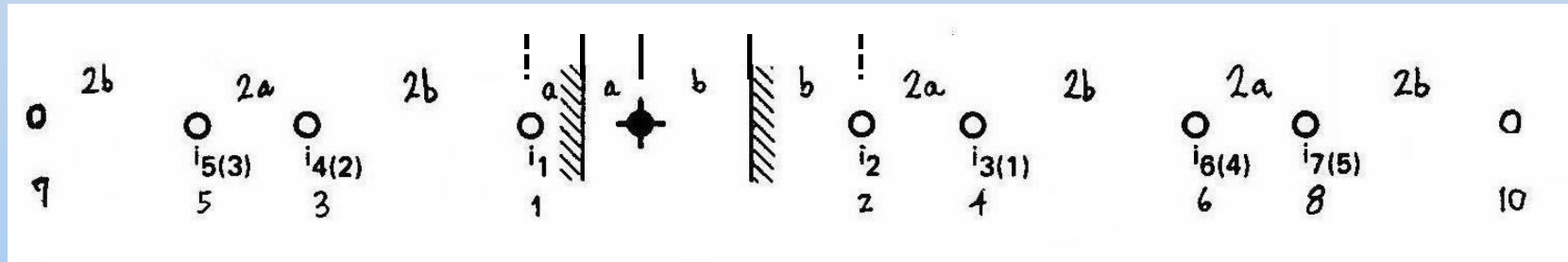
$$D_t \sim t^{1/2} \rightarrow \log[D_t] \sim \frac{1}{2} \log[t]$$

Identification of a buried valley

Log-log plot with derivative: Late time half-slope



Analysis: Image theory for two impermeable boundaries

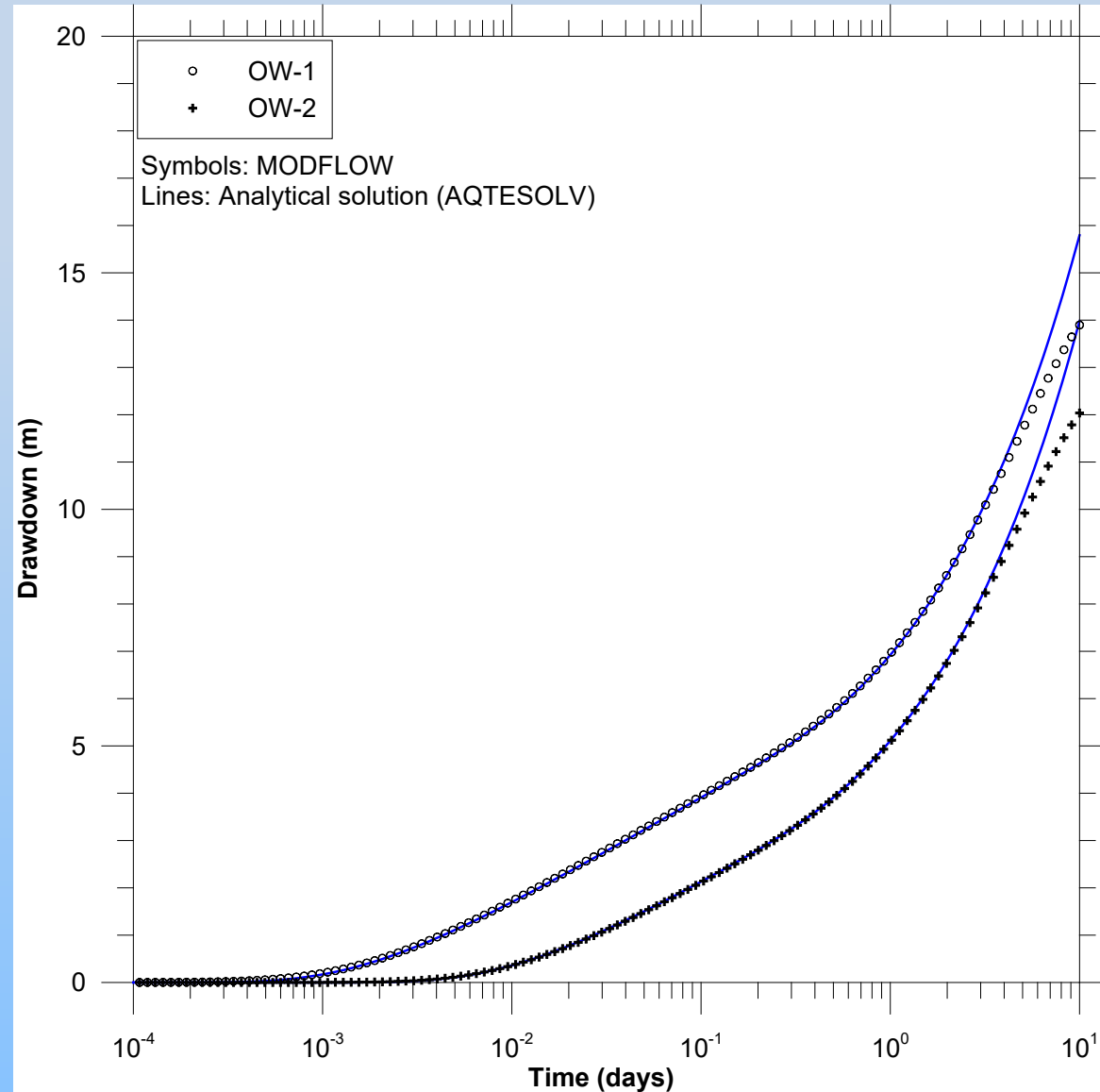


$$s(r, t) = \frac{Q}{4\pi T} \left[W(u) + \sum_{i=1}^N W(A_{ri}^2 u) \right]$$

$$A_{ri} = \frac{r_i}{r}$$

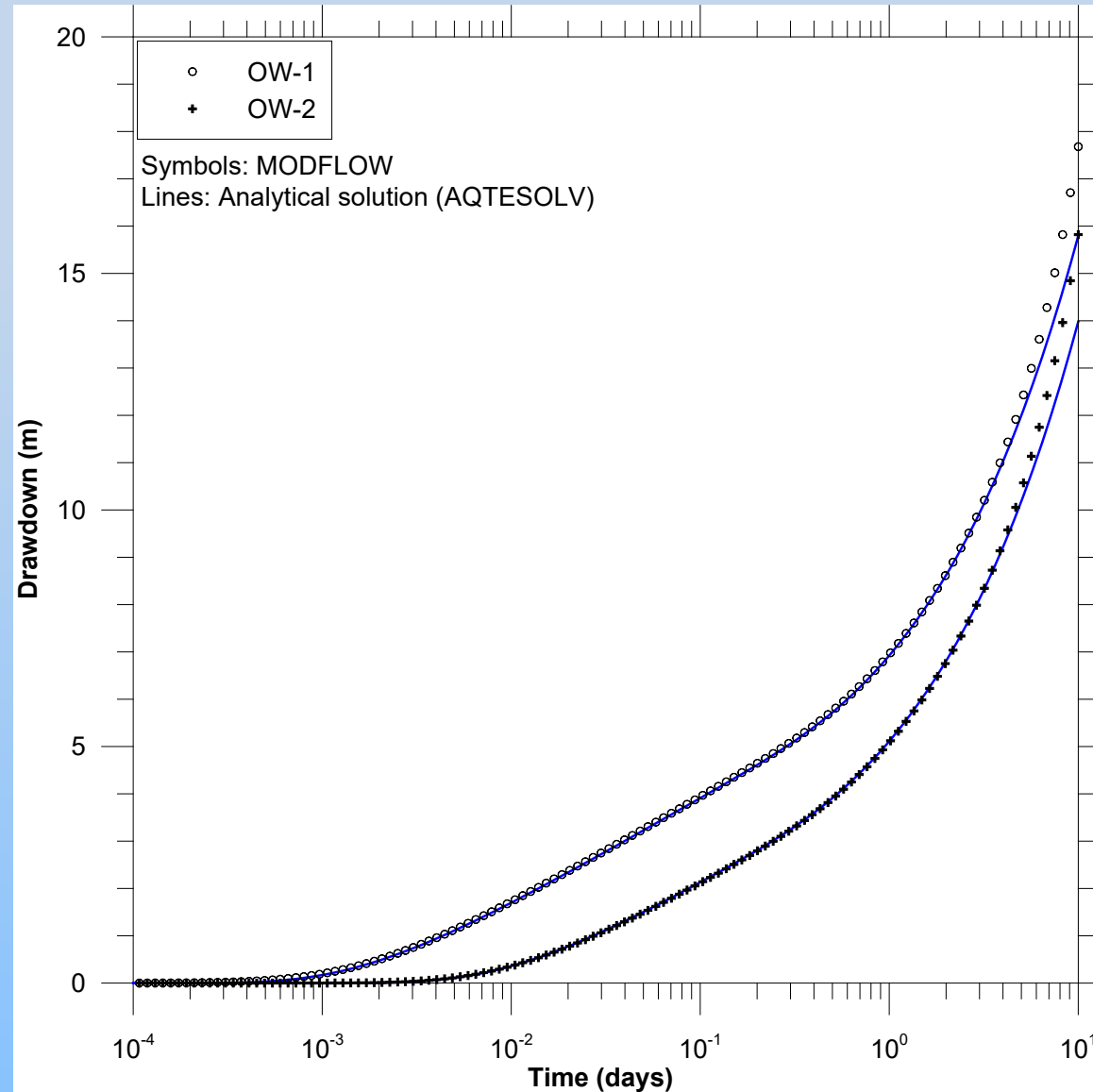
Simulation #1

Constant-head conditions
along N-S boundaries



Simulation #2

No-flow conditions along
N-S boundaries



Buried channel aquifers: Summary

1. Identify buried channel from log-log plot

Drawdown: late-time half-slope; and
Derivative: late-time half-slope.

2. Estimate transmissivity from semi-log plot

Drawdown: early-time straight line; and
Derivative: early time plateau.

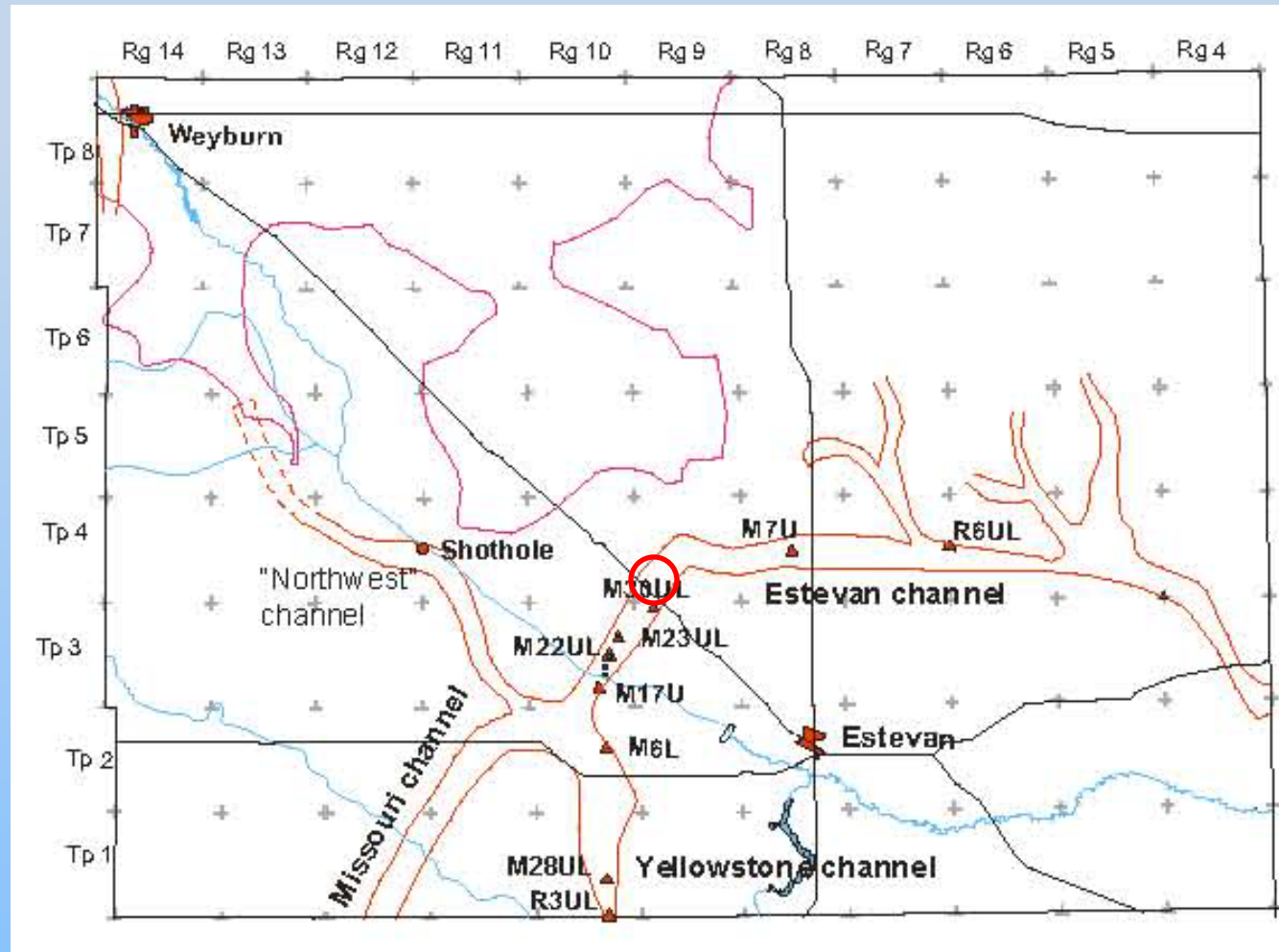
Case study Estevan, Saskatchewan (1964)





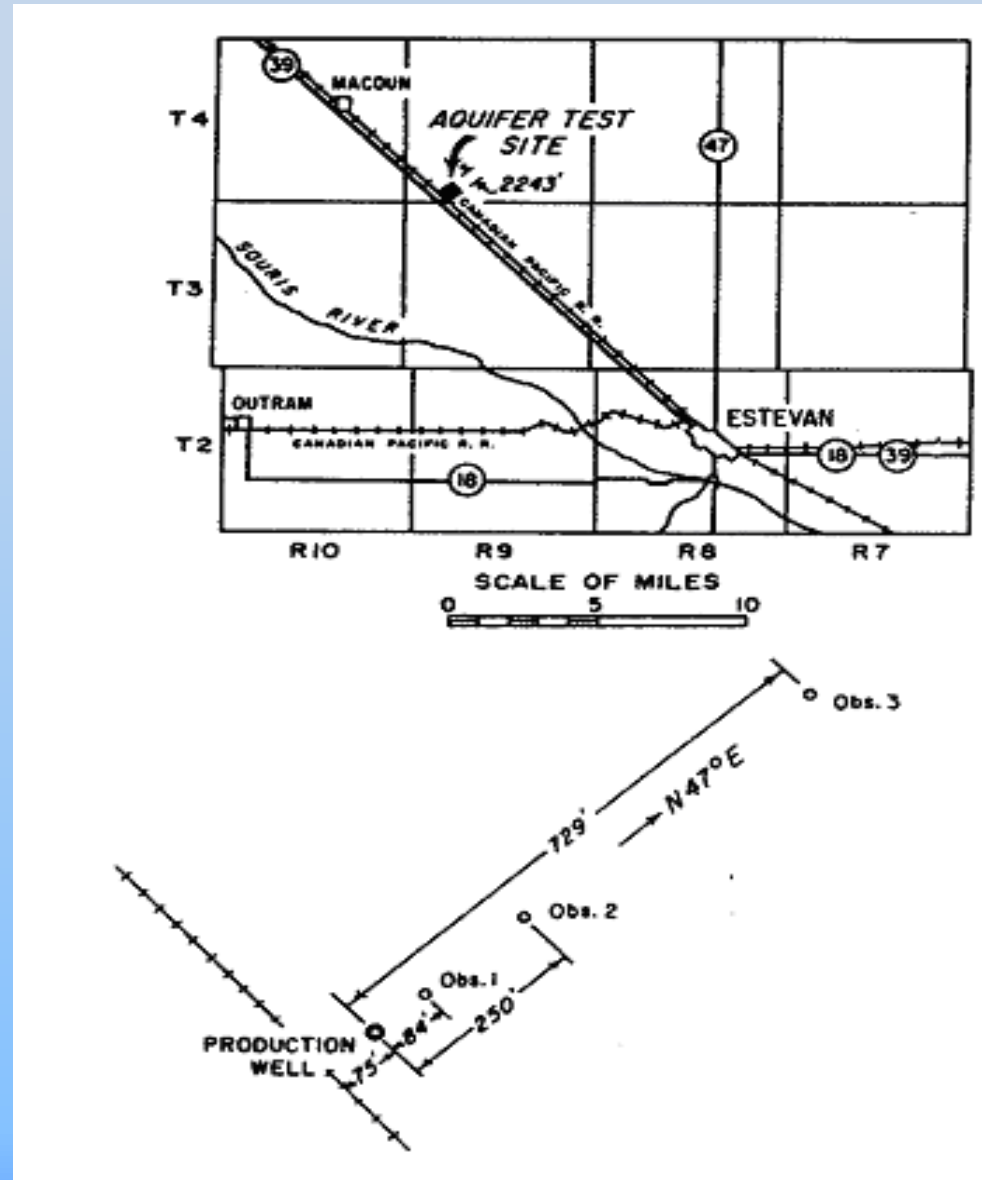
SWA Estevan Observation well, Saskatchewan

Estevan Valley aquifer system



Maathuis and van der Kamp (2003)

1964 pumping test Walton (1970)



Original analysis: Walton (1970)

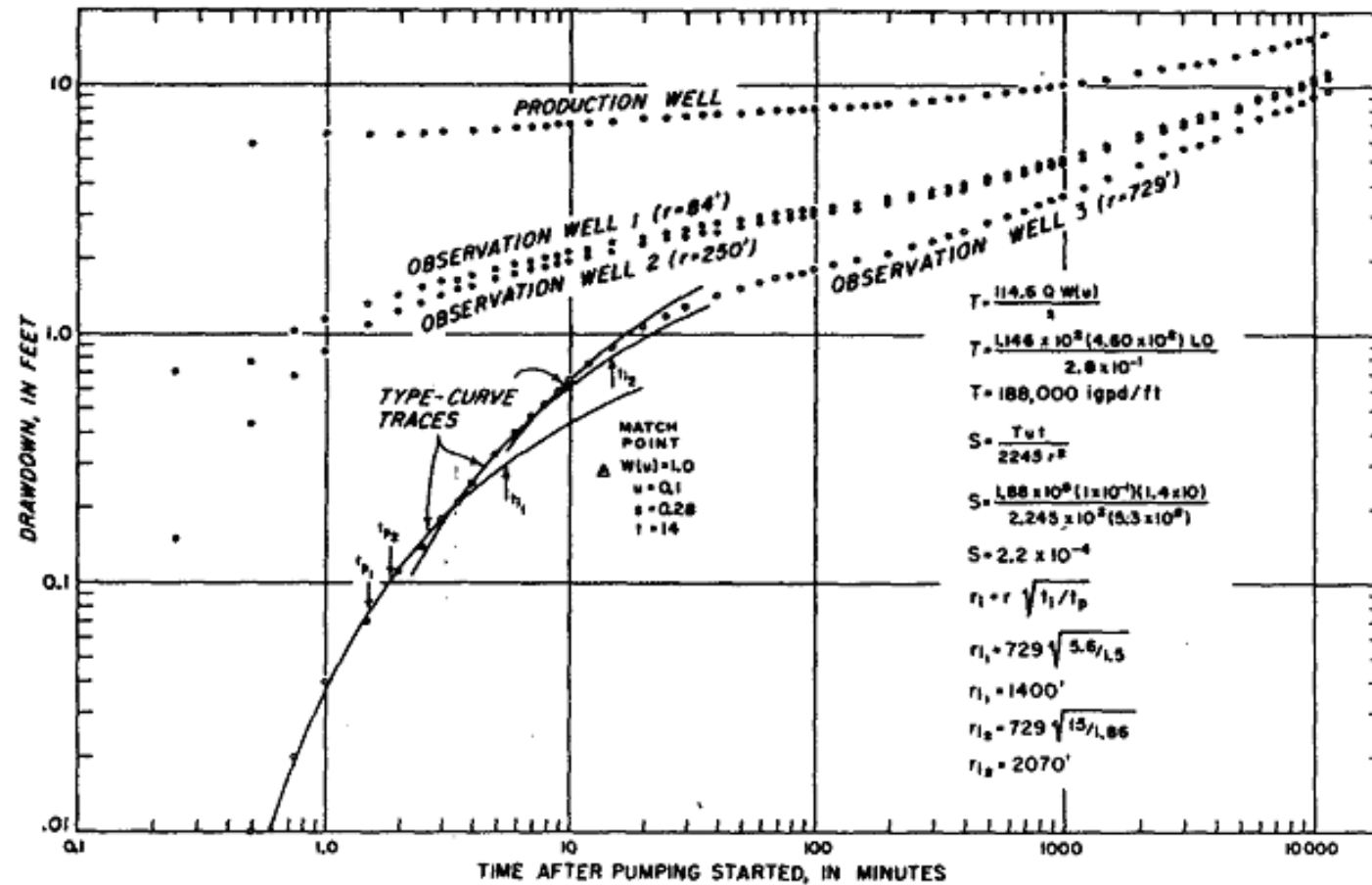


Fig. 4.42 Time-drawdown graphs for wells used in aquifer test near Estevan, Saskatchewan, Canada. (From Walton, 1965c.)

Results of Walton (1970) analysis:

$$T = 30,220 \text{ ft}^2/\text{d}$$

$$S = 2.2 \times 10^{-4}$$

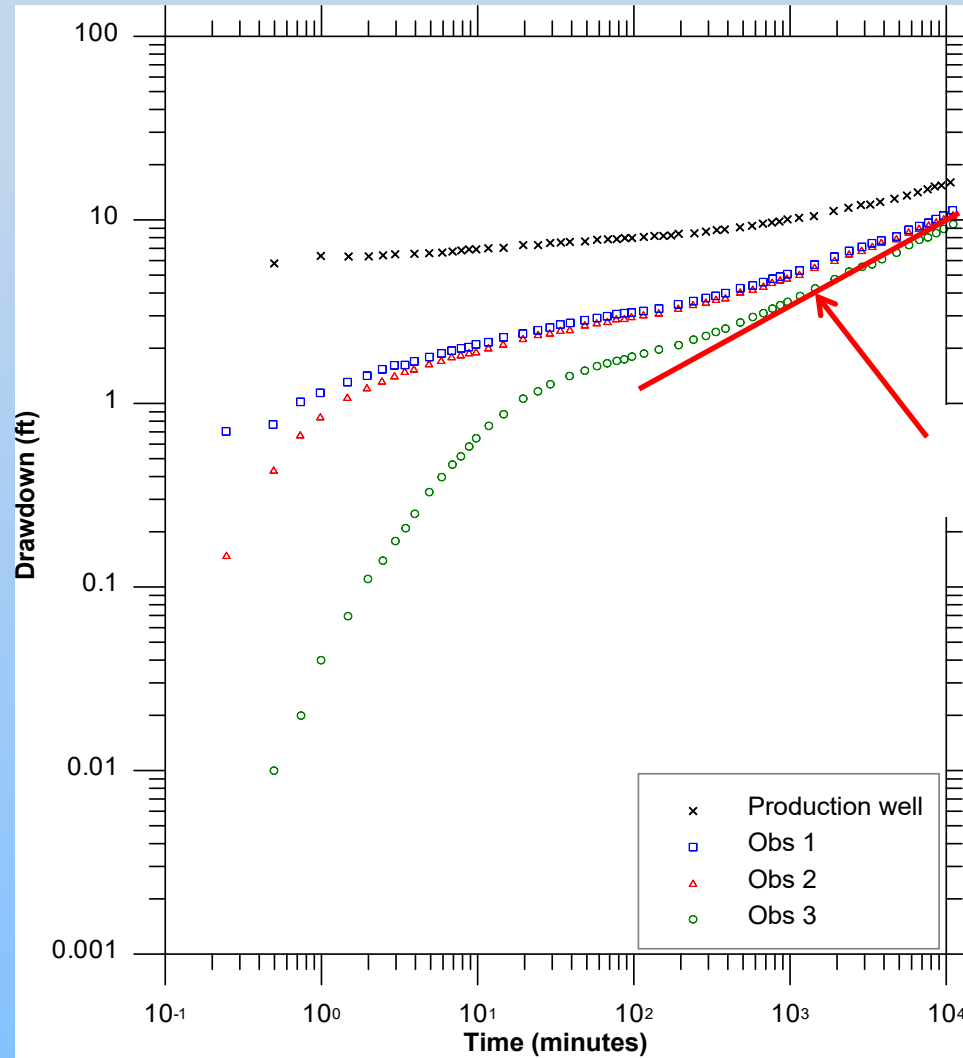
Objections:

- Recognition of the presence of boundaries, but no attempt to account for them in the analysis.
- Most of the data are neglected.
- No idea whether T is representative.

Alternative analysis

1. Diagnosis of a buried channel aquifer
2. Estimation of transmissivity from an appropriate Cooper-Jacob analysis.
 - Detection of portion of each record during which the aquifer response follows the Theis model; and
 - Estimation of bulk transmissivity using data from all wells.
3. Analysis as a buried channel aquifer

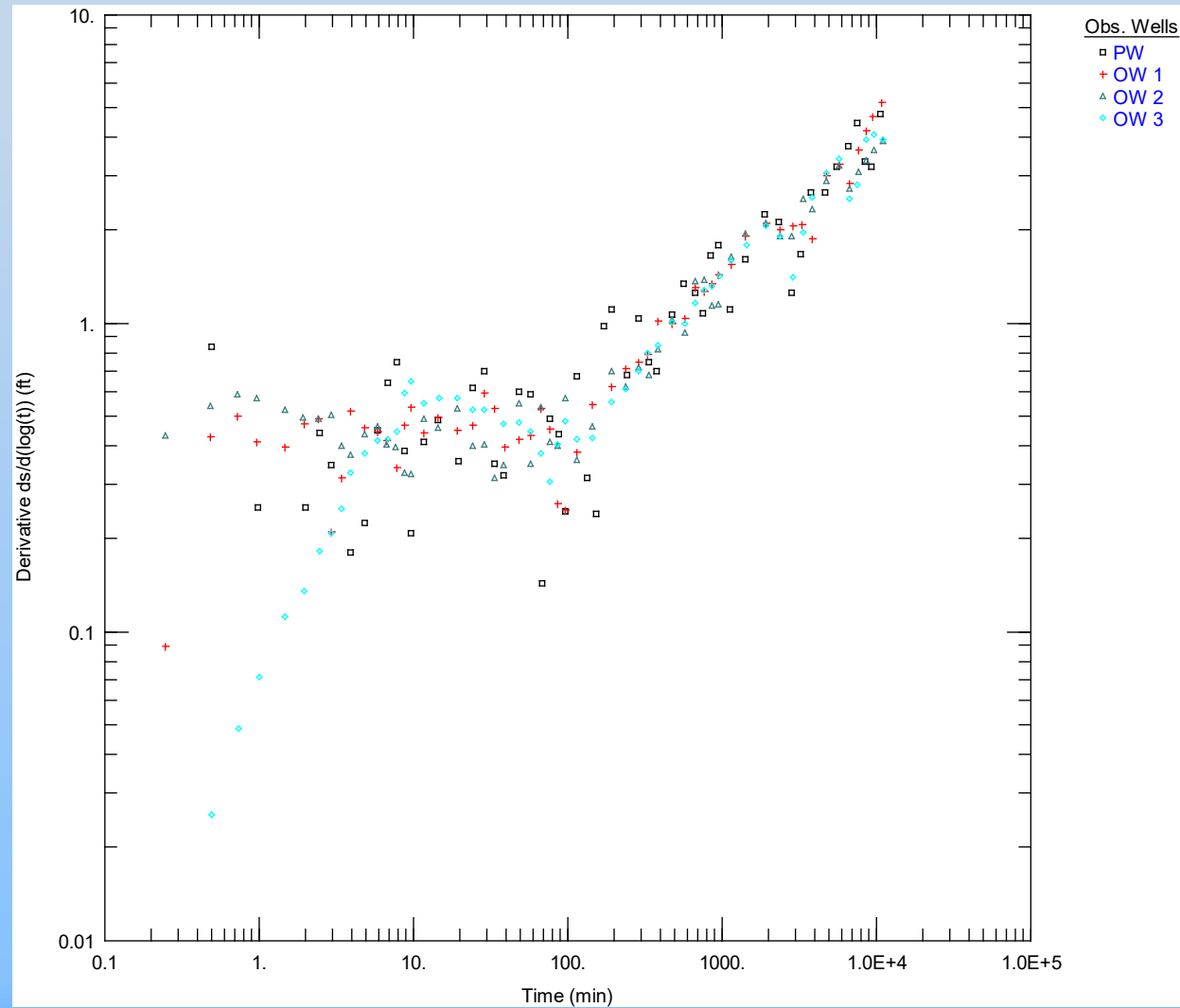
Diagnosis of buried valley: Log-log plot



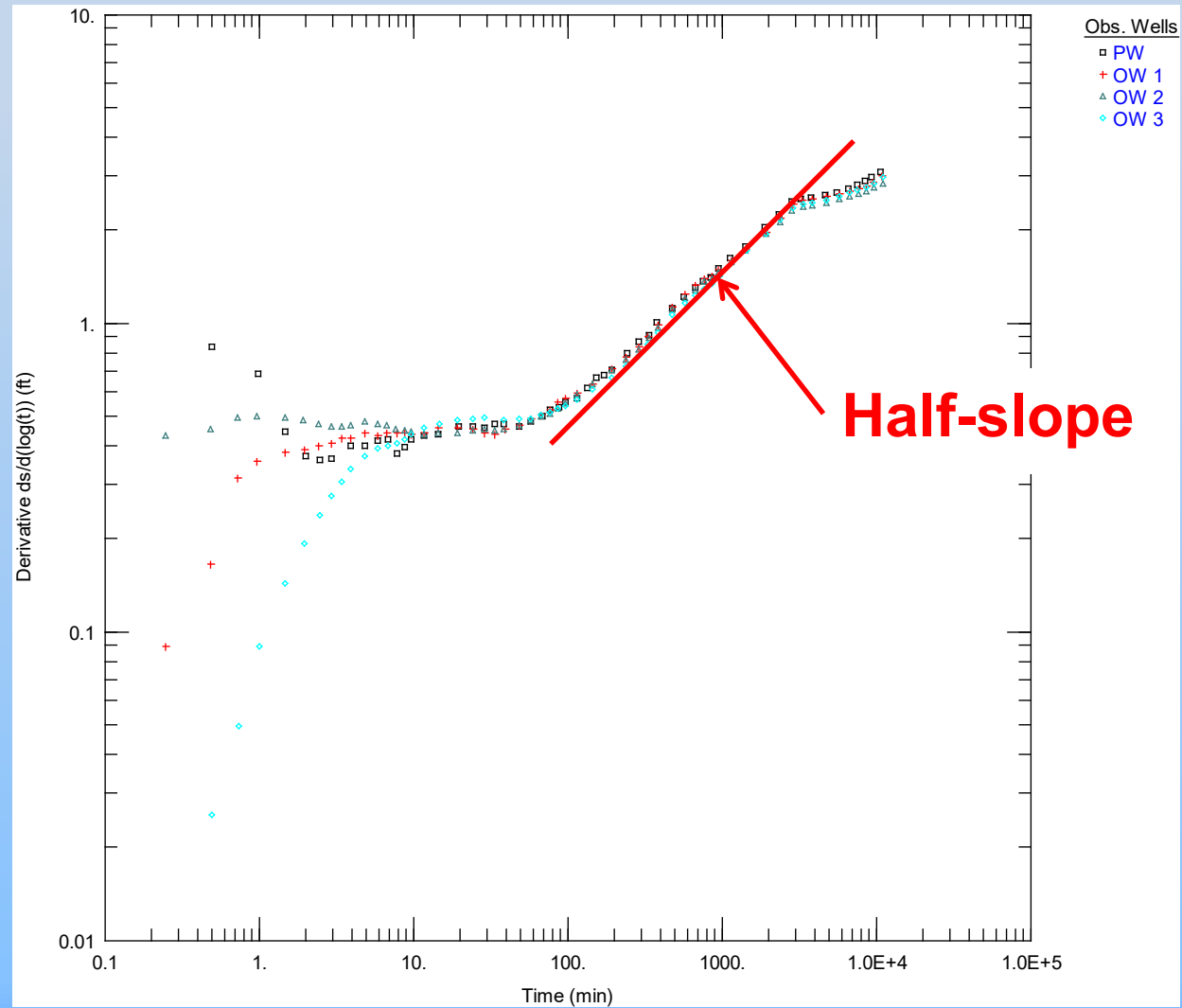
Late-time straight lines

Half-slope

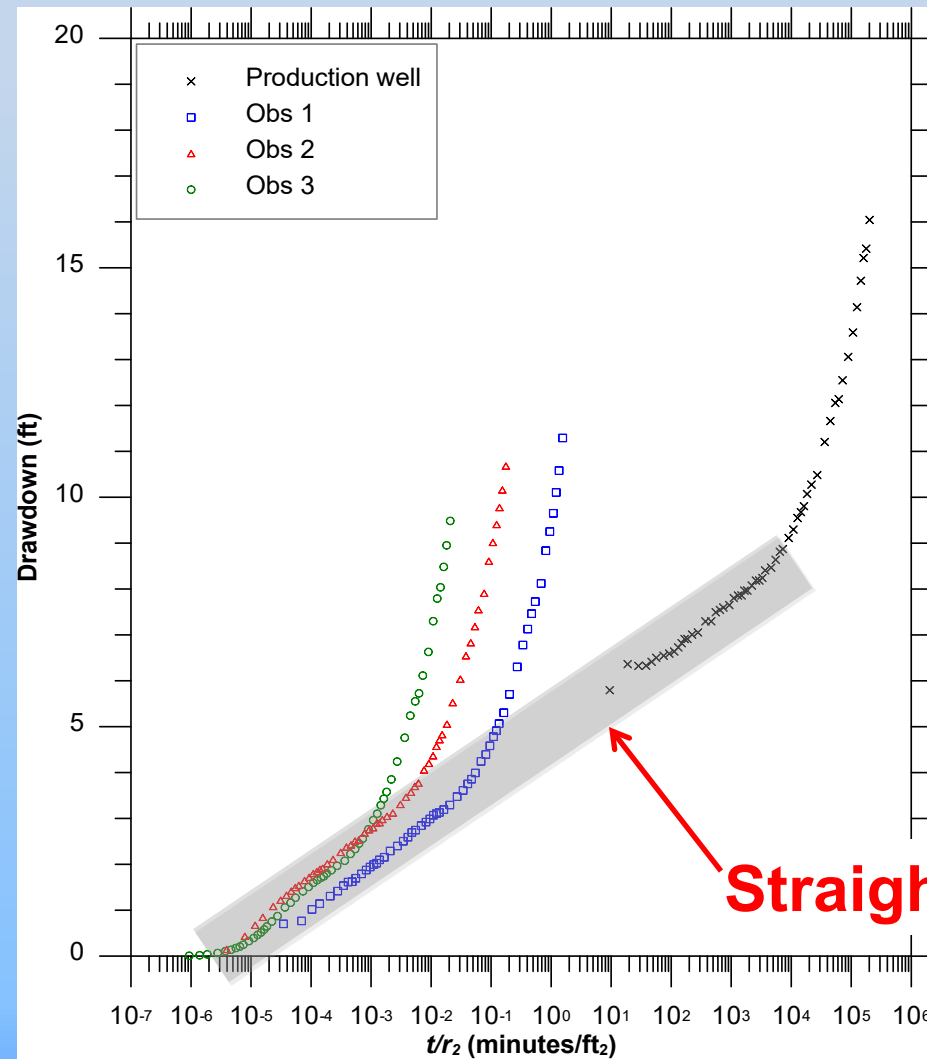
Further diagnosis: Derivative plot (raw)



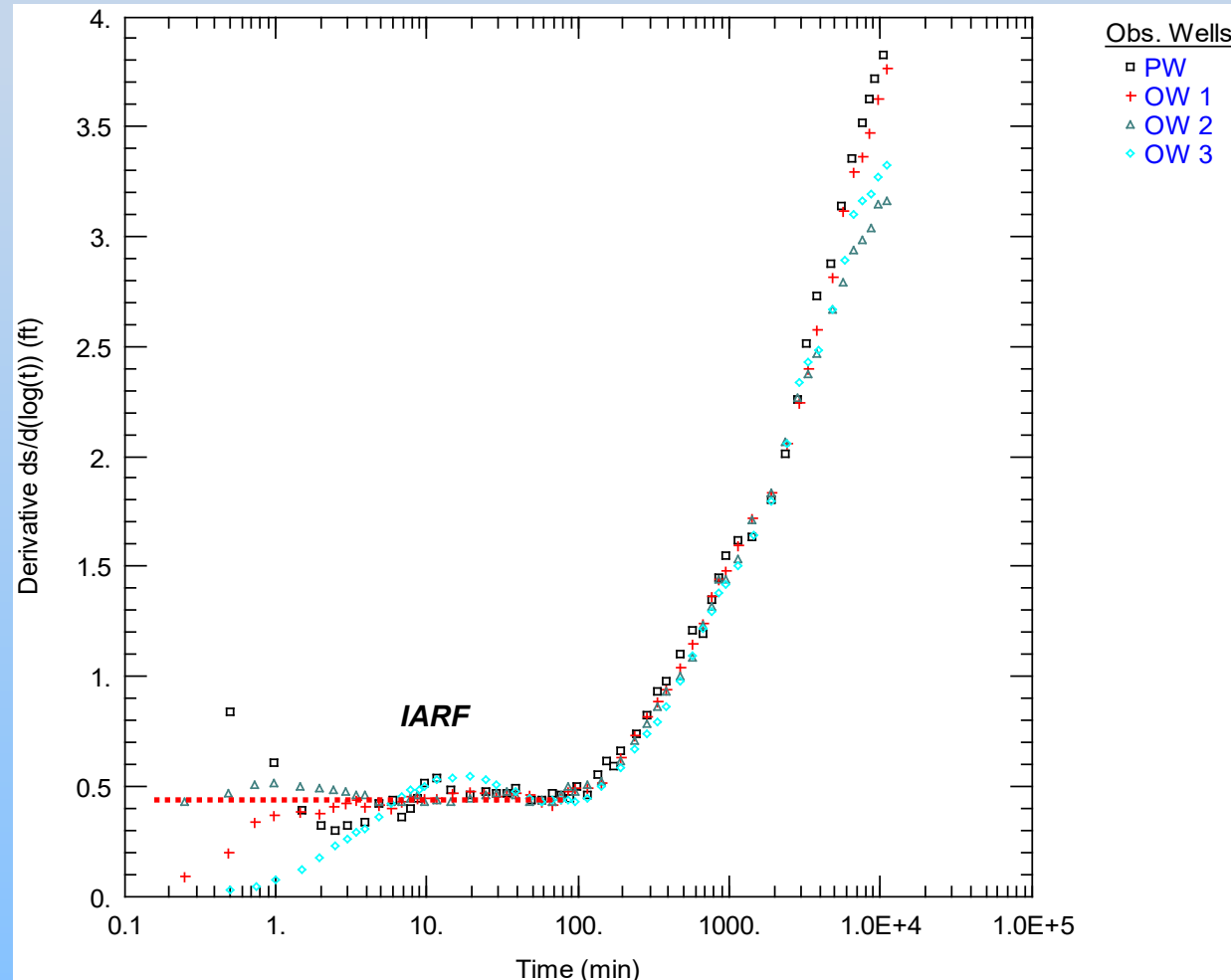
Derivative plot (smoothed)



Estimation of transmissivity: Semilog composite plot



Confirmation of the identification of the IARF period



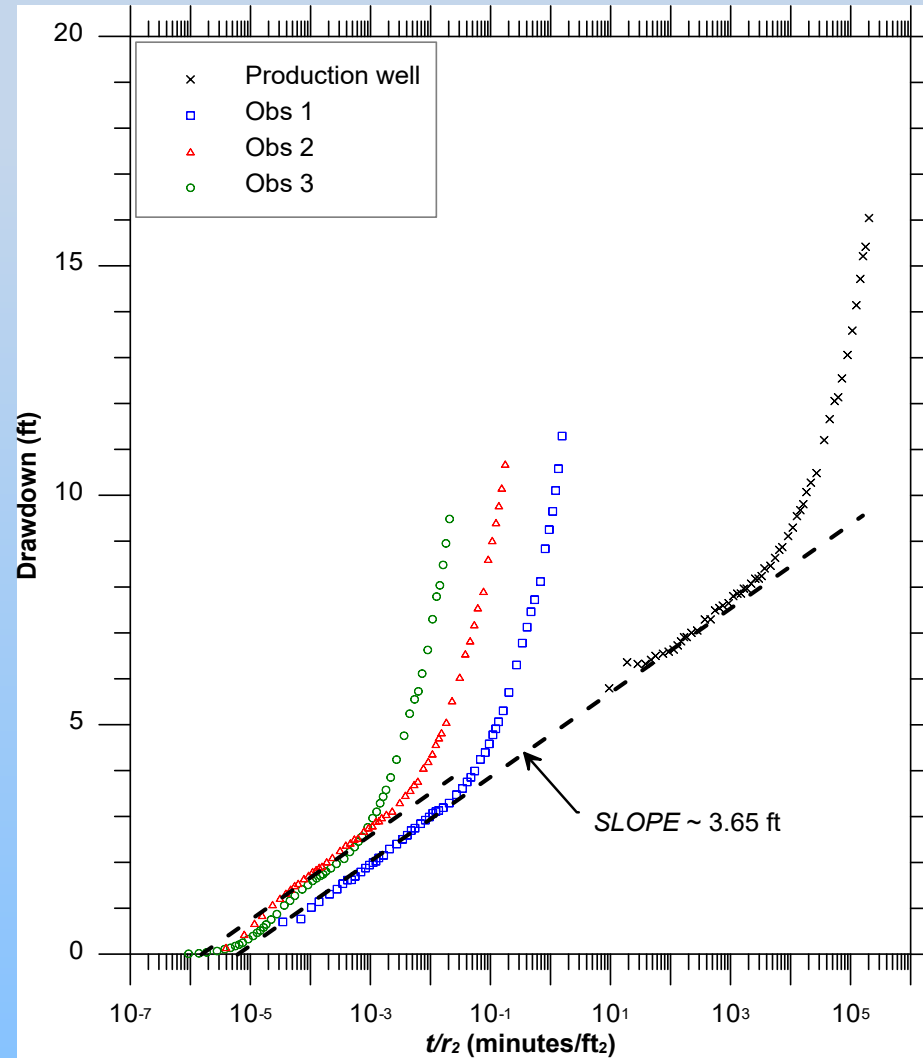
Cooper-Jacob analysis

Estimated parameters:

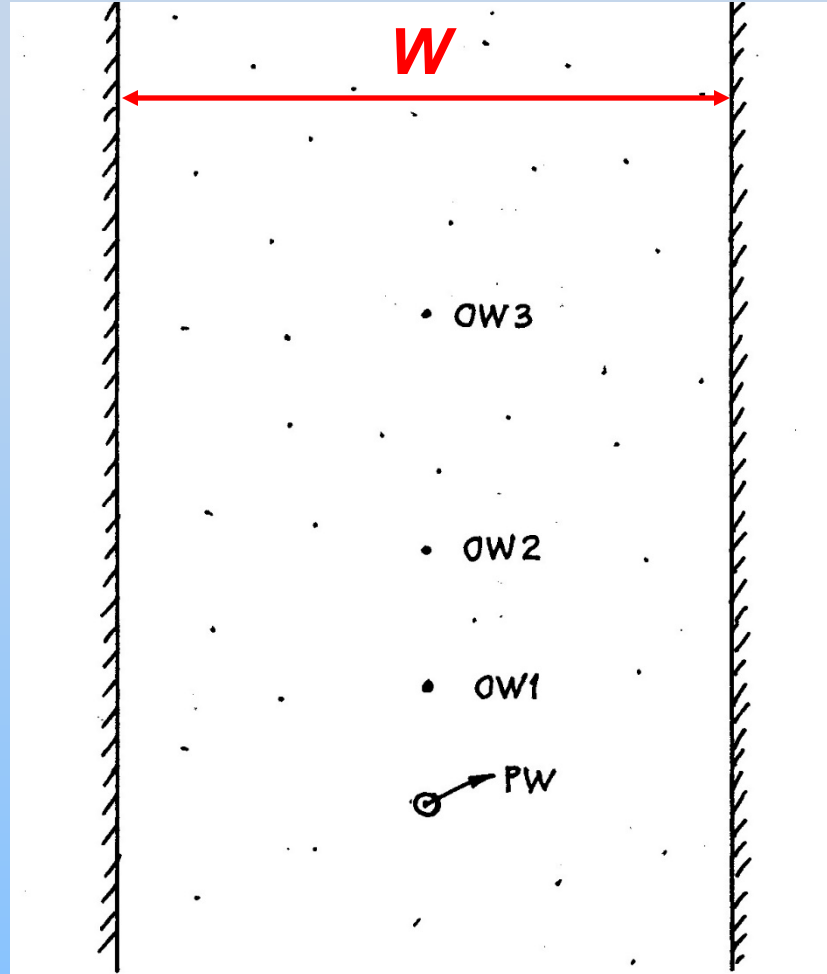
- $T = 21,000 \text{ ft}^2/\text{d}$
- $S = 1.0 \times 10^{-4}$

Walton estimates:

- $T = 30,220 \text{ ft}^2/\text{d}$
- $S = 2.2 \times 10^{-4}$



Re-analysis: Channel aquifer



Implementation of channel model in AQTESOLV

Trial #1

$W = 1000$ ft

Aquifer/Aquitard Properties

General | Aquitard | Double Porosity | Single Fracture | Wedge | Trench | Boundaries

Boundary Conditions

A-B: No Flow B-C: None

C-D: No Flow D-A: None

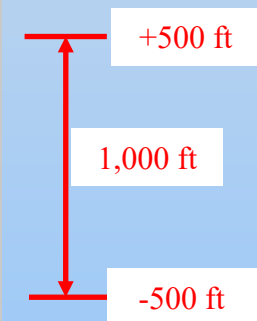
Boundary Coordinates (X,Y)

A:	-1000	500	ft
B:	1000	500	
C:	-1000	-500	
D:	1000	-500	

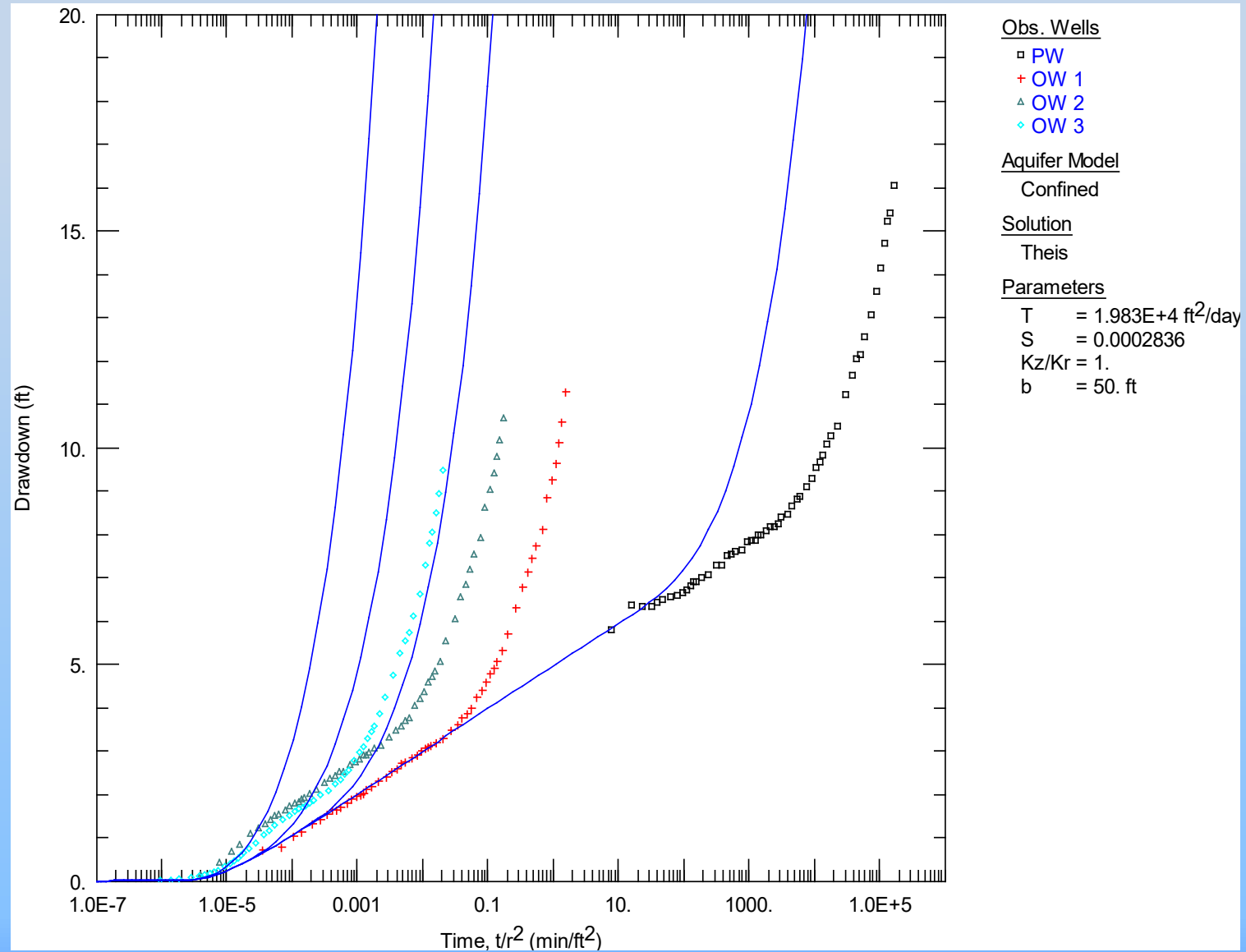
☒ Enable boundaries

Advanced

OK Cancel Apply Help



Trial #1: $W = 1,000$ ft



Trial #2

$W = 10,000$ ft

Aquifer/Aquitard Properties

General | Aquitard | Double Porosity | Single Fracture | Wedge | Trench | Boundaries

Boundary Conditions

A-B: No Flow B-C: None

C-D: No Flow D-A: None

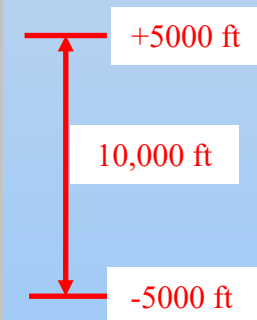
Boundary Coordinates (X,Y)

A:	-1000	5000	ft
B:	1000	5000	
C:	-1000	-5000	
D:	1000	-5000	

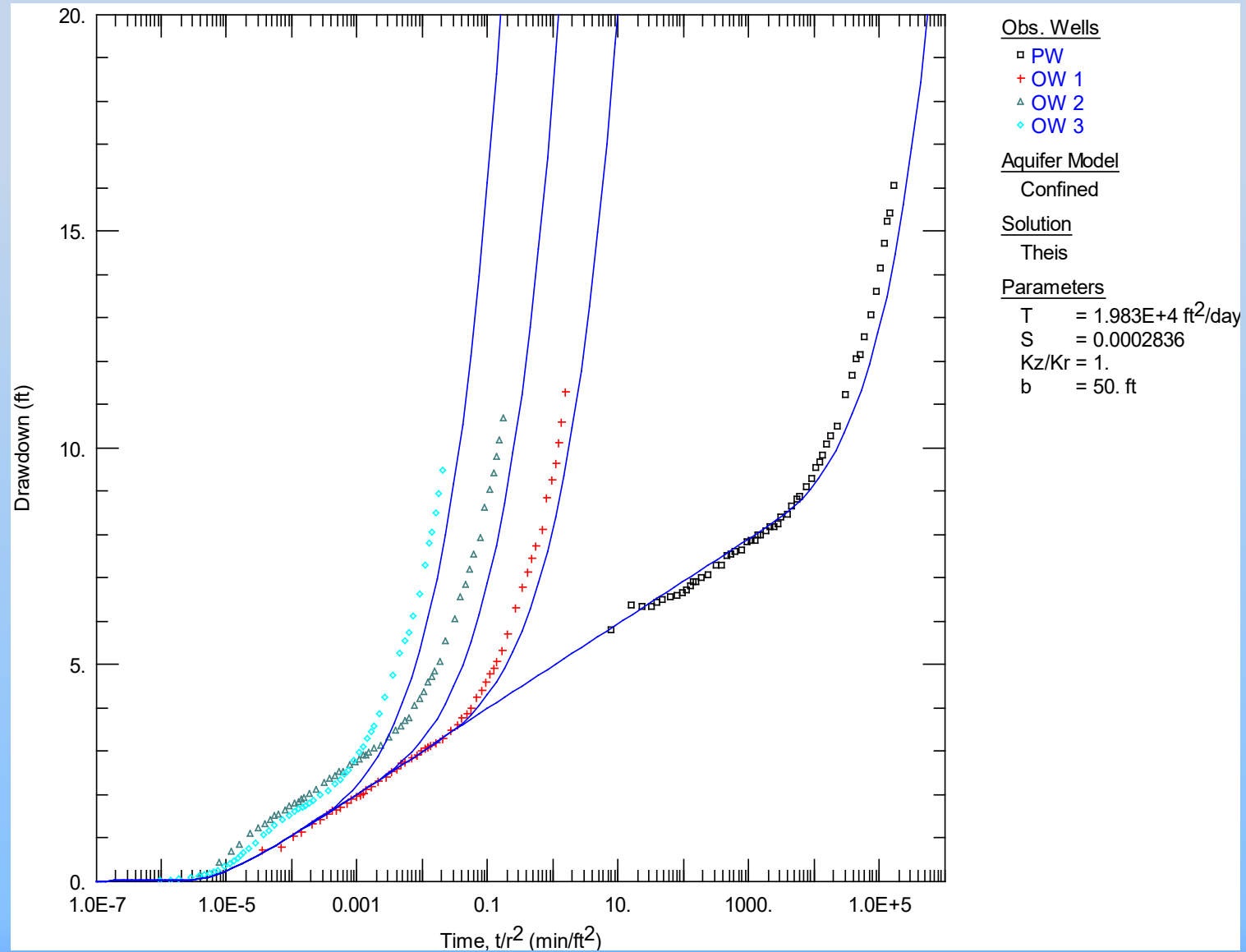
☒ Enable boundaries

Advanced

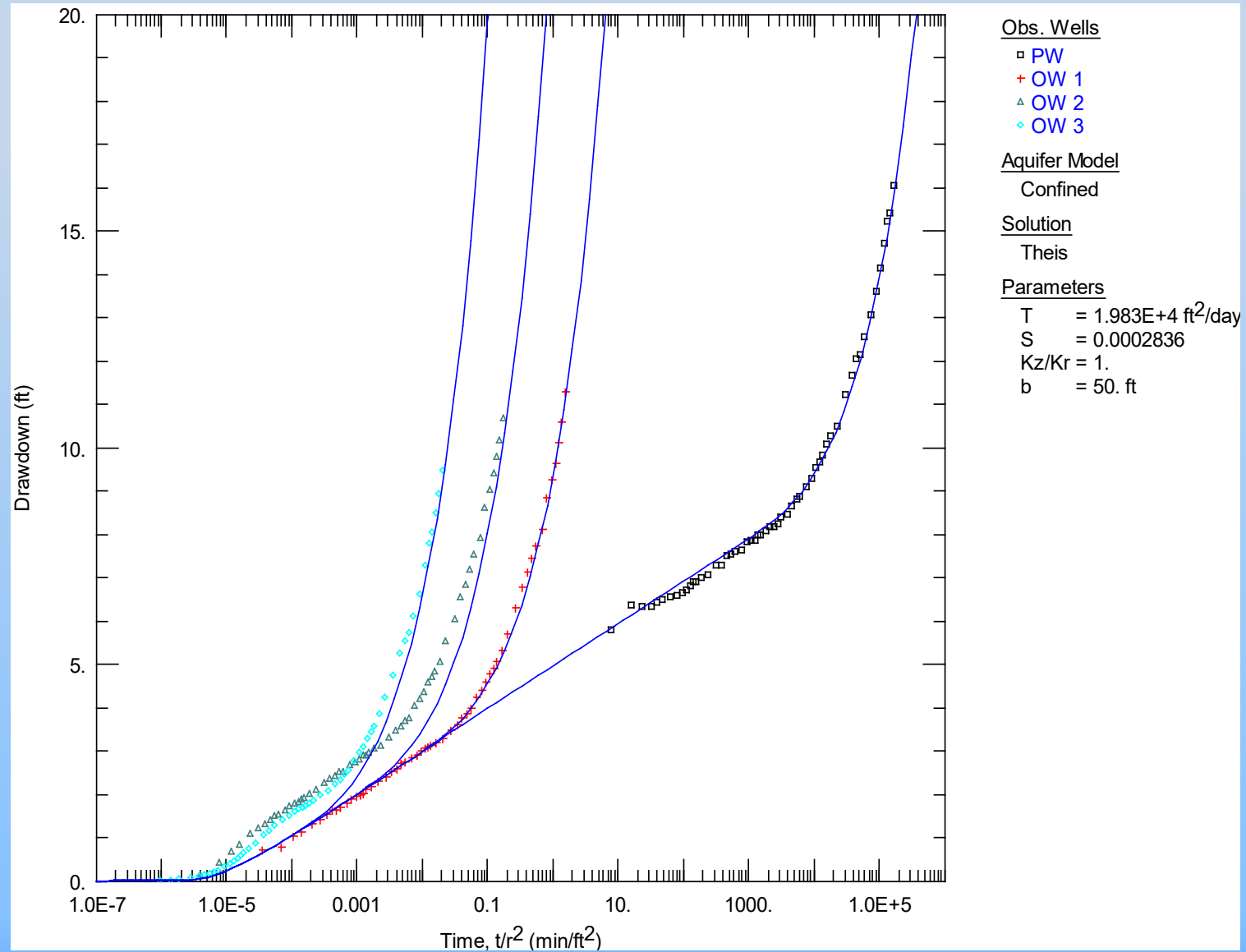
OK Cancel Apply Help



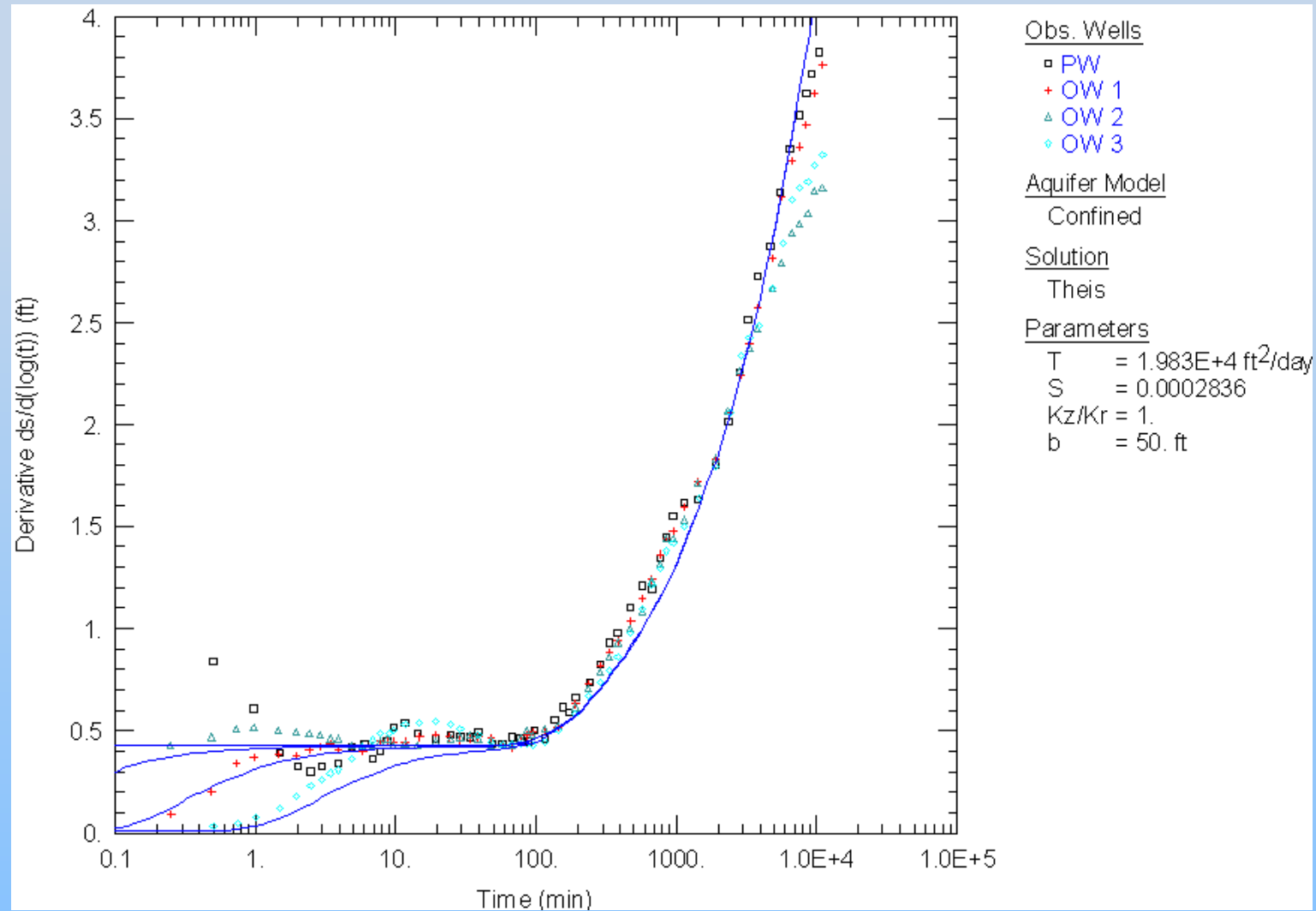
Trial #2: $W = 10,000$ ft



Trial #3: $W = 8,000$ ft



Check on conceptual model with derivative



Take-home points

1. For linear systems, boundaries can be accommodated with superposition.
2. Use both log-log and semi-log plots to enhance the diagnostic value of the analysis.
3. Plot the drawdown derivative.
4. Take advantage of computer-assisted methods to interpret pumping tests in channel aquifers.

